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Ideals, Varieties, and Algorithms

An Introduction to Computational Algebraic
Geometry and Commutative Algebra

Second Edition

With 91 Illustrations

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Contents

Preface to the First Edition	vii
Preface to the Second Edition	ix
1. Geometry, Algebra, and Algorithms	1
§1. Polynomials and Affine Space	1
§2. Affine Varieties	5
§3. Parametrizations of Affine Varieties	14
§4. Ideals	29
§5. Polynomials of One Variable	37
2. Groebner Bases	47
§1. Introduction	47
§2. Orderings on the Monomials in $k[x_1, \dots, x_n]$	52
§3. A Division Algorithm in $k[x_1, \dots, x_n]$	59
§4. Monomial Ideals and Dickson's Lemma	67
§5. The Hilbert Basis Theorem and Groebner Bases	73
§6. Properties of Groebner Bases	79
§7. Buchberger's Algorithm	86
§8. First Applications of Groebner Bases	93
§9. (Optional) Improvements on Buchberger's Algorithm	99
3. Elimination Theory	112
§1. The Elimination and Extension Theorems	112
§2. The Geometry of Elimination	120
§3. Implicitization	124
§4. Singular Points and Envelopes	133
§5. Unique Factorization and Resultants	146
§6. Resultants and the Extension Theorem	158

4. The Algebra–Geometry Dictionary	167
§1. Hilbert’s Nullstellensatz	167
§2. Radical Ideals and the Ideal–Variety Correspondence	173
§3. Sums, Products, and Intersections of Ideals	180
§4. Zariski Closure and Quotients of Ideals	190
§5. Irreducible Varieties and Prime Ideals	195
§6. Decomposition of a Variety into Irreducibles	200
§7. (Optional) Primary Decomposition of Ideals	206
§8. Summary	210
5. Polynomial and Rational Functions on a Variety	212
§1. Polynomial Mappings	212
§2. Quotients of Polynomial Rings	218
§3. Algorithmic Computations in $k[x_1, \dots, x_n]/I$	226
§4. The Coordinate Ring of an Affine Variety	235
§5. Rational Functions on a Variety	245
§6. (Optional) Proof of the Closure Theorem	254
6. Robotics and Automatic Geometric Theorem Proving	261
§1. Geometric Description of Robots	261
§2. The Forward Kinematic Problem	267
§3. The Inverse Kinematic Problem and Motion Planning	274
§4. Automatic Geometric Theorem Proving	286
§5. Wu’s Method	302
7. Invariant Theory of Finite Groups	311
§1. Symmetric Polynomials	311
§2. Finite Matrix Groups and Rings of Invariants	321
§3. Generators for the Ring of Invariants	329
§4. Relations Among Generators and the Geometry of Orbits	338
8. Projective Algebraic Geometry	349
§1. The Projective Plane	349
§2. Projective Space and Projective Varieties	360
§3. The Projective Algebra–Geometry Dictionary	370
§4. The Projective Closure of an Affine Variety	378
§5. Projective Elimination Theory	384
§6. The Geometry of Quadric Hypersurfaces	399
§7. Bezout’s Theorem	412
9. The Dimension of a Variety	429
§1. The Variety of a Monomial Ideal	429
§2. The Complement of a Monomial Ideal	433

§3. The Hilbert Function and the Dimension of a Variety	446
§4. Elementary Properties of Dimension	457
§5. Dimension and Algebraic Independence	465
§6. Dimension and Nonsingularity	473
§7. The Tangent Cone	483
Appendix A. Some Concepts from Algebra	497
§1. Fields and Rings	497
§2. Groups	498
§3. Determinants	499
Appendix B. Pseudocode	501
§1. Inputs, Outputs, Variables, and Constants	501
§2. Assignment Statements	502
§3. Looping Structures	502
§4. Branching Structures	503
Appendix C. Computer Algebra Systems	505
§1. AXIOM	505
§2. Maple	508
§3. Mathematica	510
§4. REDUCE	512
§5. Other Systems	516
Appendix D. Independent Projects	518
§1. General Comments	518
§2. Suggested Projects	518
References	523
Index	527