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Ideals, Varieties, and Algorithms

An Introduction to Computational Algebraic Geometry and Commutative Algebra

Second Edition

With 91 Illustrations

Springer
世界图书出版公司

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Mathematics Subject Classification (1991): 14-01, 13-01, 13Pxx

Library of Congress Cataloging-in-Publication Data
Cox, David A.
Ideals, varieties, and algorithms : an introduction to computational algebraic geometry and commutative algebra / by David Cox, John Little, Donal O'Shea. — 2nd ed.
p. cm.—(Undergraduate texts in mathematics)
Includes bibliographical references (p. -) and index.
1. Geometry, Algebraic—Data processing. 2. Commutative algebra—Data processing. I. Little, John B. II. O'Shea, Donal. III. Title. IV. Series.
QA564.C688 1996
516.3'5–dc20 96-8023

ISBN 978-1-4757-2695-4 ISBN 978-1-4757-2693-0 (eBook)
DOI 10.1007/978-1-4757-2693-0

9 8 7 6 5 4 3 2 1

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