

LEONHARDI EULERI OPERA OMNIA

SUB AUSPICIIS SOCIETATIS SCIENTIARUM NATURALIUM HELVETICAE

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THE RATIONAL MECHANICS

OF FLEXIBLE OR ELASTIC BODIES

1638—1788

INTRODUCTION TO

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I am happy to record that the writing was begun in the Condé-Zimmer in the Engelhof on the Nadelberg, which belonged to the BERNOULLI family during the latter part of the period described.