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Implicit Functions and Solution Mappings

A View from Variational Analysis

With 12 Illustrations



Springer

Contents

Preface	v
Acknowledgements	ix
Chapter 1. Functions defined implicitly by equations	1
1A. The classical inverse function theorem	9
1B. The classical implicit function theorem	17
1C. Calmness	21
1D. Lipschitz continuity	26
1E. Lipschitz invertibility from approximations	35
1F. Selections of multi-valued inverses	47
1G. Selections from nonstrict differentiability	51
Chapter 2. Implicit function theorems for variational problems	61
2A. Generalized equations and variational problems	62
2B. Implicit function theorems for generalized equations	74
2C. Ample parameterization and parametric robustness	83
2D. Semidifferentiable functions	88
2E. Variational inequalities with polyhedral convexity	95
2F. Variational inequalities with monotonicity	106
2G. Consequences for optimization	112
Chapter 3. Regularity properties of set-valued solution mappings	131
3A. Set convergence	134
3B. Continuity of set-valued mappings	142
3C. Lipschitz continuity of set-valued mappings	148
3D. Outer Lipschitz continuity	154

3E. Aubin property, metric regularity and linear openness	159
3F. Implicit mapping theorems with metric regularity	169
3G. Strong metric regularity	178
3H. Calmness and metric subregularity	182
3I. Strong metric subregularity	186
Chapter 4. Regularity properties through generalized derivatives	197
4A. Graphical differentiation	198
4B. Derivative criteria for the Aubin property	205
4C. Characterization of strong metric subregularity	217
4D. Applications to parameterized constraint systems	221
4E. Isolated calmness for variational inequalities	224
4F. Single-valued localizations for variational inequalities	228
4G. Special nonsmooth inverse function theorems	237
4H. Results utilizing coderivatives	245
Chapter 5. Regularity in infinite dimensions	251
5A. Openness and positively homogeneous mappings	253
5B. Mappings with closed and convex graphs	259
5C. Sublinear mappings	265
5D. The theorems of Lyusternik and Graves	274
5E. Metric regularity in metric spaces	280
5F. Strong metric regularity and implicit function theorems	292
5G. The Bartle–Graves theorem and extensions	297
Chapter 6. Applications in numerical variational analysis	311
6A. Radius theorems and conditioning	312
6B. Constraints and feasibility	320
6C. Iterative processes for generalized equations	326
6D. An implicit function theorem for Newton's iteration	336
6E. Galerkin's method for quadratic minimization	348
6F. Approximations in optimal control	352
References	363
Notation	371
Index	373