

# **Mathematics of Climate Modeling**

**Valentin P. Dymnikov  
Aleksander N. Filatov**

**Birkhäuser  
Boston • Basel • Berlin**

THIS ONE



FYQ0-E2U-XAS2

## CONTENTS

PREFACE .....	vii
INTRODUCTION .....	ix
Chapter 1. DYNAMICAL SYSTEMS. ATTRACTORS,	
INVARIANT MEASURES .....	1
1.1 Metric Spaces. Compactness .....	5
1.2 Dynamical Systems. Main Properties .....	9
1.3 Invariant Sets .....	14
1.4 Classification of Motions .....	16
1.5 Recurrence of Domains .....	23
1.6 Measure. Krylov-Bogolyubov Theorem .....	29
1.7 Dynamical Systems with Invariant Measure .....	41
1.8 Nonlinear Dissipative Systems .....	52
1.9 Inertial Manifolds of Dissipative Systems .....	67
Chapter 2. NON-AUTONOMOUS DISSIPATIVE SYSTEMS, THEIR ATTRACTOR AND AVERAGING .....	77
2.1 Introduction .....	77
2.2 Processes and their Attractors. Kernel of Processes, Section of Kernel .....	81
2.3 Families of Processes and their Attractors .....	85
2.4 Family of Processes and Semigroups .....	88
2.5 Averaging of Nonlinear Dissipative Systems. Closeness between Attractors of Original and Averaged Systems .....	93
2.6 On Closeness of Solutions of Original and Averaged Nonlinear Dissipative Systems on Infinite Time Interval .....	102
Chapter 3. ANALYSIS OF BAROTROPIC MODEL .....	109
3.1. Existence of Global Attractor .....	109
3.2 Estimate of Dimension of Attractor .....	105
3.3 Statistical Solutions and Invariant Measures on Attractor	107
3.4 Estimate of Attractor Dimension with Respect to Orography .....	121
3.5 Galerkin Approximations .....	125
3.6 Existence of Inertial Manifold .....	126
Chapter 4. DISCRETIZATION OF SYSTEMS POSSESSING ATTRACTOR .....	131
4.1 Discretization of Systems Possessing Inertial Manifolds ..	132
4.2 Time-Space Discretization of Systems Possessing Attractor	133
4.3 Globally Stable Difference Schemes for Barotropic Vorticity Equation .....	143

Chapter 5. NUMERICAL STUDY OF STRUCTURE OF ATTRACTOR GENERATED BY BAROTROPIC EQUATIONS ON SPHERE .....	171
5.1 Equations and Parameters of Model. Methods of Solving of Stationary and Nonstationary Problems .....	176
5.2 Statistical Stationary Solution and Stationary Points ....	180
5.3 Lyapunov Exponents and Attractor Dimension .....	287
5.4 Analysis of Analytical Estimates of Attractor Dimension of Barotropic Atmospheric Equations .....	188
Chapter 6. TWO-LAYER BAROCLINIC MODEL .....	189
6.1 Two-Layer Baroclinic Model .....	193
6.2 Estimate of Attractor Dimension .....	203
6.3 Numerical Investigation of Attractor. Characteristics of Two-Layer Baroclinic Model .....	209
Chapter 7. INVESTIGATION OF STRUCTURE OF CLIMATE ATTRACTORS BY OBSERVED DATA SERIES .....	211
7.1. Correlation Dimension of Attractor .....	213
7.2. Calculation of Lyapunov Exponents .....	216
7.3 Statistically Independent Degrees of Freedom and Attractor Dimension .....	217
Chapter 8. REGIMES OF ATMOSPHERE CIRCULATION ....	221
8.1 Definition of Atmosphere Circulation Regimes .....	221
8.2 Dynamical Theory of Two-Regime Barotropic Circulation	223
8.3. Statistical Theory of Two-Regime Barotropic Circulation	229
8.4 S-Regimes of Atmosphere Circulation .....	233
Chapter 9. SOLVABILITY OF OCEAN AND ATMOSPHERE MODELS .....	235
9.1 Introduction .....	235
9.2 Solvability of Ocean and Atmosphere Models in Bounded Domains .....	236
9.3 Solvability of Ocean and Atmosphere Models on Sphere in p-System of Coordinates .....	246
BIBLIOGRAPHY .....	247
INDEX .....	259