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Dynamical Systems III

Mathematical Aspects
of Classical and Celestial Mechanics

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Mathematical Aspects of Classical and Celestial Mechanics

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