

# **COMPUTATIONAL METHODS FOR PDE IN MECHANICS**

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**World Scientific**

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

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