

Series on Advances in Mathematics for Applied Sciences – Vol. 64

# **GENERALIZED KINETIC MODELS IN APPLIED SCIENCES**

**Lecture Notes on Mathematical Problem**

**Luisa Arlotti**

University of Udine, Italy

**Nicola Bellomo**

Politecnico Torino, Italy

**Elena De Angelis**

Politecnico Torino, Italy

**Mirosław Lachowicz**

Warsaw University, Poland



**World Scientific**

*New Jersey • London • Singapore • Hong Kong*

# CONTENTS

<b>Preface</b> . . . . .	<b>ix</b>
<b>Chapter 1. From the Boltzmann Equation to the Averaged Boltzmann Equation</b> . . . . .	<b>1</b>
1.1 Introduction and Motivations . . . . .	1
1.2 The Boltzmann Equation . . . . .	4
1.3 The Averaged Boltzmann Equation . . . . .	11
1.4 Modeling Dissipative Collisions . . . . .	17
1.5 The Dissipative Averaged Boltzmann Equation . . . . .	21
1.6 Generalized Kinetic (Boltzmann) Models . . . . .	24
1.7 Plan of the Lecture Notes . . . . .	27
1.8 References . . . . .	29
<b>Chapter 2. On the Cauchy Problem for the Averaged Boltzmann Equation</b> . . . . .	<b>37</b>
2.1 Introduction . . . . .	37
2.2 The Cauchy Problem for the Symmetrized Averaged Boltzmann Equation . . . . .	38
2.3 Properties of the Collision Operator . . . . .	39
2.4 The Cauchy Problem for the Symmetrized Averaged Boltzmann Equation . . . . .	50
2.5 Convergence to Equilibrium . . . . .	55
2.6 The Cauchy Problem for the Dissipative Averaged Boltzmann Equation . . . . .	63
2.7 Analysis of the Cauchy Problem . . . . .	65
2.8 Critical Analysis . . . . .	73
2.9 References . . . . .	74

<b>Chapter 3. Asymptotic Theory for the Averaged Boltzmann Equation</b>	<b>79</b>
3.1 From Microscopic to Macroscopic Description	79
3.2 Averaged Boltzmann Equation and Symmetrized Averaged Boltzmann Equation	80
3.3 Dissipative Properties of Linearized Operators	85
3.4 Stochastic Particle Systems	91
3.5 Hydrodynamic Limits for the Averaged Boltzmann Equation	106
3.6 Hydrodynamic Limits for the Symmetrized Averaged Boltzmann Equation	124
3.7 References	131
<b>Chapter 4. Kinetic (Boltzmann) Models: Modeling and Analytic Problems</b>	<b>141</b>
4.1 Introduction	141
4.2 Generalized Distribution Function	144
4.3 Modeling Microscopic Interactions	146
4.4 A Class of Evolution Equations	150
4.5 Mathematical Models	155
4.6 On the Cauchy Problem	166
4.7 Critical Analysis	179
4.8 References	182
<b>Chapter 5. Critical Analysis and Research Perspectives</b>	<b>187</b>
5.1 Introduction	187
5.2 Perspectives in Modeling	188
5.3 Perspectives on Analytic Problems	191
5.4 Concluding Remarks	192
5.5 References	193

<b>Collective Bibliography</b> . . . . .	<b>195</b>
— Asymptotic Methods in Kinetic Theory . . . . .	195
— Cauchy Problem in Kinetic Theory . . . . .	196
— Computational Methods . . . . .	196
— Kinetic Models in Applied Sciences . . . . .	196
— Mathematical Kinetic Theory . . . . .	197
— Probability Theory and Functional Analysis . . . . .	198