

Algebra

Second Edition

Volume 3

P. M. COHN, FRS

University College London

JOHN WILEY & SONS

Chichester · New York · Brisbane · Toronto · Singapore

Contents

Preface to the Second Edition	ix
Conventions on terminology and notes to the reader	xi
1 Universal algebra	
1.1 Algebras and homomorphisms	1
1.2 Congruences and the isomorphism theorems	4
1.3 Free algebras and varieties	11
1.4 Abstract dependence relations	20
1.5 The diamond lemma	25
1.6 Ultraproducts	27
1.7 The natural numbers	31
Further exercises	38
2 Multilinear algebra	
2.1 Graded algebras	40
2.2 Free algebras and tensor algebras	43
2.3 The Hilbert series of a graded algebra or module	52
2.4 The exterior algebra on a module	58
Further exercises	66
3 Homological algebra	
3.1 Additive and abelian categories	69
3.2 Functors on abelian categories	78
3.3 The category $\mathcal{M}_R$	87
3.4 Homological dimension	94
3.5 Derived functors	99
3.6 Ext and Tor and the global dimension of a ring	110
Further exercises	120
4 Further group theory	
4.1 Group extensions	124
4.2 The Frattini subgroup and the Fitting subgroup	135

4.3	Hall subgroups	141
4.4	The transfer	146
4.5	Free groups	148
4.6	Commutators	154
4.7	Linear groups	159
	Further exercises	165
5	Further field theory	
5.1	Algebraic dependence	168
5.2	Simple transcendental extensions	171
5.3	Separable and p -radical extensions	176
5.4	Derivations	181
5.5	Linearly disjoint extensions	186
5.6	Infinite algebraic extensions	196
5.7	Galois cohomology	202
5.8	Kummer extensions	206
	Further exercises	210
6	Algebras	
6.1	The Krull-Schmidt theorem	213
6.2	The projective cover of a module	218
6.3	Semiperfect rings	221
6.4	Equivalence of module categories	227
6.5	The Morita context	234
6.6	Projective, injective and flat modules	239
6.7	Hochschild cohomology and separable algebras	248
	Further exercises	256
7	Central simple algebras	
7.1	Simple Artinian rings	258
7.2	The Brauer group	266
7.3	The reduced norm and trace	273
7.4	Quaternion algebras	280
7.5	Crossed products	283
7.6	Change of base field	289
7.7	Cyclic algebras	295
	Further exercises	298
8	Quadratic forms and ordered fields	
8.1	The Clifford algebra of a quadratic space	300
8.2	The spinor norm	306
8.3	Formally real fields	309
8.4	The Witt ring of a field	322

8.5	The symplectic group	330
8.6	The orthogonal group	337
8.7	Quadratic forms in characteristic 2	344
	Further exercises	347
9	Noetherian rings and polynomial identities	
9.1	Rings of fractions	349
9.2	Principal ideal domains	355
9.3	Skew polynomials and Laurent series	359
9.4	Goldie's theorem	367
9.5	PI-algebras	375
9.6	Varieties of PI-algebras and Regev's theorem	380
9.7	Generic matrix rings and central polynomials	384
9.8	Generalized polynomial identities	389
	Further exercises	393
10	Rings without finiteness assumptions	
10.1	The endomorphism ring of a vector space	395
10.2	The density theorem revisited	398
10.3	Primitive rings	401
10.4	Semiprimitive rings and the Jacobson radical	404
10.5	Algebras without a unit element	409
10.6	Semiprime rings and nilradicals	413
10.7	Prime PI-algebras	419
10.8	Simple algebras with minimal right ideals	423
10.9	Firs and semifirs	425
	Further exercises	430
11	Skew fields	
11.1	Generalities	432
11.2	The Dieudonné determinant	436
11.3	Free fields	443
11.4	Valuations on skew fields	448
11.5	Pseudo-linear extensions	455
	Further exercises	459
	Bibliography	461
	List of notations	464
	Index	468