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The Classification of the Finite Simple Groups, Number 3

Part I, Chapter A: Almost Simple \mathcal{K} -Groups

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