

A Practical Guide to
**BOUNDARY
ELEMENT METHODS**
with the Software Library BEMLIB

C. POZRIKIDIS



CHAPMAN & HALL/CRC

A CRC Press Company

Boca Raton London New York Washington, D.C.

Contents

Frequently Asked Questions	xi
1 Laplace's equation in one dimension	1
1.1 Green's first and second identities and the reciprocal relation	1
1.2 Green's functions	2
1.3 Boundary-value representation	7
1.4 Boundary-value equation	8
2 Laplace's equation in two dimensions	9
2.1 Green's first and second identities and the reciprocal relation	9
2.2 Green's functions	12
2.3 Integral representation	19
2.4 Integral equations	28
2.5 Hypersingular integrals	36
2.6 Irrotational flow	44
2.7 Generalized single- and double-layer representations	47
3 Boundary-element methods for Laplace's equation in two dimensions	53
3.1 Boundary element discretization	53
3.2 Discretization of the integral representation	63
3.3 The boundary-element collocation method	74
3.4 Isoparametric cubic-splines discretization	77
3.5 High-order collocation methods	80
3.6 Galerkin and global expansion methods	86
4 Laplace's equation in three dimensions	91
4.1 Green's first and second identities and the reciprocal relation	91
4.2 Green's functions	93
4.3 Integral representation	100
4.4 Integral equations	102
4.5 Axisymmetric fields in axisymmetric domains	106
5 Boundary-element methods for Laplace's equation in three dimensions	111
5.1 Discretization	111
5.2 Three-node flat triangles	117
5.3 Six-node curved triangles	122
5.4 High-order expansions	127

6	Inhomogeneous, nonlinear, and time-dependent problems	131
6.1	Distributed source and domain integrals	132
6.2	Particular solutions and dual reciprocity in one dimension	134
6.3	Particular solutions and dual reciprocity in two and three dimensions	141
6.4	Convection – diffusion equation	150
6.5	Time-dependent problems	154
7	Viscous flow	161
7.1	Governing equations	161
7.2	Stokes flow	165
7.3	Boundary integral equations in two dimensions	171
7.4	Boundary-integral equations in three dimensions	175
7.5	Boundary-element methods	178
7.6	Interfacial dynamics	180
7.7	Unsteady, Navier-Stokes, and non-Newtonian flow	182
8	BEMLIB user guide	191
8.1	General information	191
8.2	Terms and conditions	192
8.3	Directory contents	193
9	Directory: grids	197
	grid_2d	198
	trgl	203
10	Directory: laplace	207
	lgf_2d	209
	lgf_3d	230
	lgf_ax	245
	flow_1d	254
	flow_1d_lp	261
	flow_2d	265
	body_2d	270
	body_ax	276
	tank_2d	281
	ldr_3d	287
	lrm_3d	291
11	Directory: helmholtz	295
	flow_1d_osc	296
12	Directory: stokes	301
	sgf_2d	302
	sgf_3d	329
	sgf_ax	349
	flow_2d	364

prtclsw	371
prtcl2d	376
prtclax	385
prtcl3d	390
Appendix A Mathematical supplement	397
Appendix B Gauss elimination and linear solvers	403
Appendix C Elastostatics	407
References	413
Index	418