

David Salomon

Curves and Surfaces for Computer Graphics

With 207 Figures, 12 in Full Color



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ISBN-10: 0-387-24196-5
ISBN-13: 978-0-387-24196-8

e-ISBN: 0-387-28452-4

Printed on acid-free paper.

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Printed in the United States of America. (HAM)

9 8 7 6 5 4 3 2 1 SPIN 11360285

springeronline.com

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My kind publishers announced, some time ago, a table of contents, which included chapters on jay and fish-hawk, panther, and musquash, and a certain savage old bull moose that once took up his abode too near my camp for comfort. My only excuse for their non-appearance is that my little book was full before their turn came.

They will find their place, I trust, in another volume presently.

—William J. Long, *Secret of the Woods* (1901)