

Roman Mikhailov • Inder Bir Singh Passi

Lower Central and Dimension Series of Groups

 Springer

Contents

1	Lower Central Series	1
1.1	Commutator Calculus	1
1.2	Residually Nilpotent Groups	2
1.3	Generalized Relation Modules	17
1.4	k -central Extensions	37
1.5	Nilpotent Completion	66
1.6	Bousfield-Kan Completion	71
1.7	Homological Localization	75
1.8	Crossed Modules and Cat^1 -Groups	84
2	Dimension Subgroups	101
2.1	Groups Without Dimension Property	101
2.2	Sjögren's Theorem	119
2.3	Fourth Dimension Subgroup	121
2.4	Fifth Dimension Subgroup	123
2.5	Quasi-varieties of Groups	134
2.6	The Quasi-variety \mathcal{D}_4	137
2.7	Dimension Quotients	143
2.8	Plotkin's Problems	148
2.9	Modular Dimension Subgroups	151
2.10	Lie Dimension Subgroups	152
2.11	Lie Nilpotency Indices	160
2.12	Subgroups Dual to Dimension Subgroups	163
3	Derived Series	165
3.1	Commutator Subgroups of Free Nilpotent Groups	165
3.2	E and D -groups	171
3.3	Transfinite Derived Series	176
3.4	Applications to Asphericity	179

4	Augmentation Powers	187
4.1	Augmentation Identities	187
4.2	Integral Augmentation Powers	187
4.3	Intersection Theorems	190
4.4	Transfinite Augmentation Powers	191
4.5	Schur Multiplier	196
4.6	Relative Dimension Subgroups	210
4.7	A Characterization of Para-free Groups	214
4.8	r -para-free Groups	215
4.9	Homological Localization of $\mathbb{Z}[G]$ -modules	219
5	Homotopical Aspects	229
5.1	The Associated Graded Ring of a Group Ring	229
5.2	Spectral Sequences	233
5.3	Applications to Dimension Subgroups	243
5.4	Homotopical Applications	254
5.5	Computations and Connectivity Results	272
6	Miscellanea	291
6.1	Power-closed Groups	291
6.2	Braid Groups	291
6.3	3-dimensional Surgery	293
6.4	Vanishing Sums of Roots of Unity	295
6.5	Fundamental Groups of Projective Curves	296
A	Simplicial Methods	299
A.1	Chain Complexes	299
A.2	Simplicial Objects	300
A.3	Geometric Realization Functor	302
A.4	Skeleton and Coskeleton Functors	302
A.5	Moore Complex and Homotopy Groups	304
A.6	Dold-Kan Correspondence	306
A.7	Eilenberg-Zilber Equivalence	307
A.8	Classifying Functor \overline{W} and Homology	308
A.9	Bisimplicial Groups	309
A.10	Certain Simplicial Constructions	310
A.11	Free Simplicial Resolutions	314
A.12	Functorial Properties	317
A.13	Derived Functors	320
A.14	Quadratic Functors	322
A.15	Derived Functors in the Sense of Dold and Puppe	326
A.16	Derived Limits and Fibration Sequence	328
	References	331
	Index	343