

The Matrix Eigenvalue Problem

***GR* and Krylov Subspace Methods**

David S. Watkins

Washington State University
Pullman, Washington

Contents

Preface	ix
1 Preliminary Material	1
1.1 Matrix Algebra	1
1.2 Norms and Inner Products	3
1.3 Matrix Groups and Decompositions	8
1.4 Some Other Classes of Matrices	15
1.5 Subspaces	18
1.6 Projectors	25
1.7 The Singular Value Decomposition (SVD)	27
1.8 Linear Transformations and Matrices	30
2 Basic Theory of Eigensystems	33
2.1 Eigenvalues, Eigenvectors, and Invariant Subspaces	33
2.2 Similarity, Unitary Similarity, and Schur's Theorem	41
2.3 The Spectral Theorem for Normal Matrices	47
2.4 The Jordan Canonical Form	52
2.5 Spectral Projectors	71
2.6 Angles and Distances between Subspaces	74
2.7 Perturbation Theorems	90
3 Elimination	115
3.1 Matrices That Create Zeros in Vectors	115
3.2 Creating Zeros in Matrices, <i>GR</i> Decompositions	126
3.3 Similarity Transformation to Upper Hessenberg Form	136
3.4 Working from Bottom to Top	145
4 Iteration	151
4.1 Subspace Iteration	151
4.2 Simultaneous Iteration and <i>GR</i> Algorithms; <i>QR</i> Algorithm	154
4.3 Shifting to Accelerate Convergence	160
4.4 The Differential Quotient-Difference Algorithm	165
4.5 Economical <i>GR</i> Iterations by Bulge-Chasing	171
4.6 The Singular Case	180

4.7	A Closer Look at Bulge-Chasing	185
4.8	Computing Eigenvectors and Invariant Subspaces	190
4.9	Practical Bulge-Chasing Algorithms	198
5	Convergence	213
5.1	Convergence of Subspace Iteration	213
5.2	Convergence of <i>GR</i> Algorithms	221
6	The Generalized Eigenvalue Problem	233
6.1	Introduction	233
6.2	Reduction to Hessenberg-Triangular Form	237
6.3	<i>GZ</i> Algorithms	241
6.4	The <i>HZ</i> Algorithm	247
6.5	Infinite Eigenvalues	249
6.6	Deflating Subspaces	256
7	Inside the Bulge	265
7.1	Transmission of Shifts	265
7.2	Bulge-Chasing in Reverse	276
7.3	Passing Bulges through Each Other	284
8	Product Eigenvalue Problems	291
8.1	Introduction	291
8.2	The <i>GR</i> Algorithm for a Product of Matrices	293
8.3	Implicit Product <i>GR</i> Iterations	296
8.4	The Singular Value Decomposition	303
8.5	Accurate Computation of Singular Values	308
8.6	Eigenvalue Problems Related to the SVD	315
8.7	The Generalized Eigenvalue Problem	317
8.8	The Hamiltonian Eigenvalue Problem	319
8.9	Pseudosymmetric, Hamiltonian, and Symplectic Matrices	337
8.10	The Unitary Eigenvalue Problem	341
8.11	The Totally Nonnegative Eigenvalue Problem	346
9	Krylov Subspace Methods	349
9.1	Introduction	349
9.2	The Generic Krylov Process	357
9.3	Implicit Restarts	362
9.4	The Arnoldi and Symmetric Lanczos Processes	370
9.5	The Unitary Arnoldi Process	380
9.6	The Unsymmetric Lanczos Process	388
9.7	Skew-Hamiltonian Krylov Processes	398
9.8	The Hamiltonian Lanczos Process	403
9.9	The Symplectic Lanczos Process	406
9.10	Product Krylov Processes	410
9.11	Block Krylov Processes	416

Contents	vii
Bibliography	421
Index	437