Gilbert G. Walter
Martha Contreras

Compartmental Modeling with Networks

Birkhäuser
Boston • Basel • Berlin

Contents

Preface ix

List of Figures xviii

1 Introduction and Simple Examples 1
1.1 Mathematical Models . . . 1
1.2 Examples of Models . . . 2

Part I. Structure of Models: Directed Graphs 9

2 Digraphs and Graphs: Definitions and Examples 11
2.1 Definitions . . . 11
2.2 Examples . . . 12
2.3 Problems . . . 15

3 A Little Simple Graph Theory 17
3.1 Isomorphic Graphs and Digraphs . . . 17
3.2 Connected Graphs and Digraphs . . . 19

4 Orientation of Graphs and Related Properties 25
4.1 Vertex Basis . . . 25
4.2 Multigraphs . . . 27
4.3 Orientation of Graphs . . . 31
4.4 Spanning Trees . . . 33
4.5 Minimum Connector Problem . . . 37

5 Tournaments 41
5.1 Definitions and Basic Results . . . 41
5.2 Transitive Tournaments . . . 44

6 Planar Graphs 47
6.1 Bipartite Graphs . . . 48
6.2 A Necessary Condition for a Graph to Be Planar . . . 49

7 Graphs and Matrices 53
7.1 Adjacency and Reachability Matrices 53
7.2 Eigenvalues of Adjacency Matrices 56
7.3 Using Maple with Graphs 59

Part II. Digraphs and Probabilities: Markov Chains 63

8 Introduction to Markov Chains 65
8.1 Relation to Digraphs 65
8.2 More Definitions and Examples 66

9 Classification of Markov Chains 71
9.1 Definitions 71
9.2 Condensation of a Stochastic Digraph 76

10 Regular Markov Chains 81
10.1 Theory of Regular Chains 81
10.2 Fixed-Point Probability Vector 84
10.3 Influence Digraph 86

11 Absorbing Markov Chains 89
11.1 An Example 89
11.2 Some General Results 92
11.3 Population Genetics: An Example of an Absorbing Chain . 95
11.4 Small Group Decision Making: An Absorbing Markov Chain 97

12 From Markov Chains to Compartmental Models 101
12.1 Comparison of Quantities 101
12.2 Two Examples 104

Part III. Compartmental Models: Applications 109

13 Introduction to Compartmental Models 111
13.1 Basic Concepts 111
13.2 One-Compartment Applications 114
13.2.1 Linear Case 115
13.2.2 Logistic Growth 116
13.3 Other Examples 118
13.4 Matrix Forms 120
13.5 Two-Compartment Models 121

14 Models for the Spread of Epidemics 125
14.1 The SIR Model 125
14.2 Other Models of Epidemics 128

15 Three Traditional Examples as Compartmental Models 131
15.1 Predator–Prey or Host–Parasite Equations 131
15.2 The Leslie Matrix 135
15.3 Leontief Input–Output Analysis 137

16 Ecosystem Models 141
16.1 Dissolved Oxygen 141
16.2 Forest Ecosystem 143
16.3 Food Webs 146

17 Fisheries Models 149
17.1 Logistic Equation Approach 150
17.2 Nonequilibrium Yield 152
17.3 A Multispecies Model 156
17.4 An Ecosystem Fisheries Model 159

18 Drug Kinetics 163
18.1 Bilirubin Metabolism (Simple) 163
18.2 Bilirubin Metabolism (Complex) 165
18.3 Lead Kinetics 166
18.4 HIV Dynamics 167

Part IV. Compartmental Models Theory 173

19 Basic Properties of Linear Models 175
19.1 Compartmental Matrices 176
19.2 Eigenvalues 178
19.3 Analytic Solution 180

20 Structure and Dynamical Properties 183
20.1 Positivity of Solutions 183
20.2 Condensation of the Digraph 185
20.3 Eigenvalues and Structure 189
20.4 Some Special Cases 192

21 Identifiability of a Compartmental System 197
21.1 General Input and Output 197
21.2 An Example of an Identifiable System 199
21.3 Another Example Which Is Not Identifiable 200
21.4 Another Approach: Using Laplace Transforms 201
21.5 The General Case 203
21.6 Size Identifiability of Compartmental Models 204
21.6.1 Example 206
21.6.2 Some More Examples 207
21.6.3 Main Result 209

22 Parameter Estimation **211**
22.1 Estimation Problem 211
22.2 Statistical Estimation 215

23 Complexity and Stability **218**
23.1 Complexity 218
23.2 The Shannon Index 221
23.3 Other Indices 223
23.4 Relation to Stability 226

A Mathematical Prerequisites **228**

Appendix **228**
A.1 Matrix Operations 228
A.2 Finding Eigenvalues and Eigenvectors 233
A.3 Systems of Differential Equations 235
A.4 Matrices with Maple 238

Bibliography **242**

Index **247**