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SPDE in Hydrodynamic: Recent Progress and Prospects

Lectures given at the
C.I.M.E. Summer School
held in Cetraro, Italy
August 29–September 3, 2005

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 Springer



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Some Methods of Infinite Dimensional Analysis

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