Grothendieck Duality and Base Change



Contents

| Chapter 1. Introduction | 1 |
|---|-----|
| 1.1. Overview and Motivation | 1 |
| 1.2. Notation and Terminology | |
| 1.3. Sign Conventions | 7 |
| Chapter 2. Basic Compatibilities | 21 |
| 2.1. General Nonsense | 21 |
| 2.2. Smooth and Finite Maps | 28 |
| 2.3. Projective Space and the Trace Map | 32 |
| 2.4. Proofs of Properties of the Projective Trace | 30 |
| 2.5. The Fundamental Local Isomorphism | 52 |
| 2.6. Proofs of Properties of the Fundamental Local Isomorphism | 56 |
| Compatibilities between (·)^t and (·)^b | 76 |
| Gluing (·)^t and (·)^b | 99 |
| Chapter 3. Duality Foundations | 105 |
| 3.1. Dualizing Complexes | 105 |
| 3.2. Residual Complexes | 125 |
| 3.3. The Functor (·)! and Residual Complexes | 133 |
| 3.4. The Trace Map Tr _f and Grothendieck-Serre Duality | 146 |
| 3.5. Dualizing Sheaves and CM maps | 153 |
| 3.6. Base Change for Dualizing Sheaves | 164 |
| Chapter 4. Proof of Main Theorem | 175 |
| 4.1. Case of an Artinian Quotient | 175 |
| 4.2. Case of Artin Local Base Schemes | 182 |
| 4.3. Duality for Proper CM Maps in the Locally Noetherian Case | 189 |
| 4.4. Conclusion of Proof of Duality Theorem | 201 |
| Chapter 5. Examples | 217 |
| 5.1. Higher Direct Images | 217 |
| 5.2. Curves | 225 |
| Appendix A. Residues and Cohomology with Supports | 237 |
| A.1. Statement of Results | 237 |
| A.2. Proofs | 241 |
| Appendix B. Trace Man on Smooth Curves | 971 |

CONTENTS

| B.1. | Motivation | 271 |
|--------------|----------------------|-----|
| B.2. | Preparations | 277 |
| B.3. | The Proof | 277 |
| B.4. | Duality on Jacobians | 283 |
| Bibliography | | 291 |
| Index | | 293 |