

Jerrold E. Marsden · Gerard Misiolek
Juan-Pablo Ortega · Matthew Perlmutter
Tudor S. Ratiu

Hamiltonian Reduction by Stages

 Springer

Contents

Part I: Background and the Problem Setting	1
1 Symplectic Reduction	3
1.1 Introduction to Symplectic Reduction	3
1.2 Symplectic Reduction: Proofs and Further Details	12
1.3 Reduction Theory: Historical Overview	24
1.4 Overview of Singular Symplectic Reduction	36
2 Cotangent Bundle Reduction	43
2.1 Principal Bundles and Connections	43
2.2 Cotangent Bundle Reduction: Embedding Version	50
2.3 Cotangent Bundle Reduction: Bundle Version	71
2.4 Singular Cotangent Bundle Reduction	88
3 The Problem Setting	101
3.1 The Setting for Reduction by Stages	101
3.2 Applications and Infinite Dimensional Problems	106
Part II: Regular Symplectic Reduction by Stages	111
4 Commuting Reduction and Semidirect Product Theory	113
4.1 Commuting Reduction	113
4.2 Semidirect Products	119

4.3	Cotangent Bundle Reduction and Semidirect Products	132
4.4	Example: The Euclidean Group	137
5	Regular Reduction by Stages	143
5.1	Motivating Example: The Heisenberg Group	144
5.2	Point Reduction by Stages	149
5.3	Poisson and Orbit Reduction by Stages	171
6	Group Extensions and the Stages Hypothesis	177
6.1	Lie Group and Lie Algebra Extensions	178
6.2	Central Extensions	198
6.3	Group Extensions Satisfy the Stages Hypotheses	201
6.4	The Semidirect Product of Two Groups	204
7	Magnetic Cotangent Bundle Reduction	211
7.1	Embedding Magnetic Cotangent Bundle Reduction	212
7.2	Magnetic Lie-Poisson and Orbit Reduction	225
8	Stages and Conjoint Orbits of Central Extensions	239
8.1	Stage One Reduction for Central Extensions	240
8.2	Reduction by Stages for Central Extensions	245
9	Examples	251
9.1	The Heisenberg Group Revisited	252
9.2	A Central Extension of $L(S^1)$	253
9.3	The Oscillator Group	259
9.4	Bon-Vincent Group	267
9.5	Fluids with a Spatial Symmetry	279
10	Stages and Semidirect Products with Cocycles	285
10.1	Abelian Semidirect Product Extensions	
	First Reduction	286
10.2	Abelian Semidirect Product Extensions	
	Conjoint Orbits	295
10.3	Coupling to a Lie Group	304
10.4	Poisson Reduction by Stages	
	General Semidirect Products	309
	First Stage Reduction: General Semidirect Products	315
10.6	Second Stage Reduction: General Semidirect Products	321
10.7	Example: The Group $T \ltimes \mathfrak{so}(d)$	347
11	Reduction by Stages via Symplectic Distributions	397
11.1	Reduction by Stages of Connected Components	398
11.2	Momentum Level Sets and Distributions	401
11.3	Proof: Reduction by Stages II	406

12 Reduction by Stages with Topological Conditions	409
12.1 Reduction by Stages III	409
12.2 Relation Between Stages II and III	416
12.3 Connected Components of Reduced Spaces	419
Conclusions for Part I	420
Part III: Optimal Reduction and Singular Reduction by Stages, by Juan-Pablo Ortega	421
13 The Optimal Momentum Map and Point Reduction	423
13.1 Optimal Momentum Map and Space	423
13.2 Momentum Level Sets and Associated Isotropies	426
13.3 Optimal Momentum Map Dual Pair	427
13.4 Dual Pairs, Reduced Spaces, and Symplectic Leaves	430
13.5 Optimal Point Reduction	432
13.6 The Symplectic Case and Sjamaar's Principle	435
14 Optimal Orbit Reduction	437
14.1 The Space for Optimal Orbit Reduction	437
14.2 The Symplectic Orbit Reduction Quotient	443
14.3 The Polar Reduced Spaces	446
14.4 Symplectic Leaves and the Reduction Diagram	454
14.5 Orbit Reduction: Beyond Compact Groups	455
14.6 Examples: Polar Reduction of the Conjugate Action	457
15 Optimal Reduction by Stages	461
15.1 The Polar Distribution of a Normal Subgroup	461
15.2 Isotropy Subgroups and Quotient Groups	464
15.3 The Optimal Reduction by Stages Theorem	466
15.4 Optimal Orbit Reduction by Stages	470
15.5 Reduction by Stages of Globally Hamiltonian Actions	475
Acknowledgments for Part III	481
Bibliography	483
Index	509