
LINEAR OPERATORS

PART II:

SPECTRAL THEORY

Self Adjoint Operators in Hilbert Space

NELSON DUNFORD and JACOB T. SCHWARTZ

FORMER JAMES E. ENGLISH PROFESSOR OF MATHEMATICS
PROFESSOR OF MATHEMATICS COURANT INSTITUTE
YALE UNIVERSITY NEW YORK UNIVERSITY

WITH THE ASSISTANCE OF

William G. Bade and Robert G. Bartle
PROFESSOR OF MATHEMATICS PROFESSOR OF MATHEMATICS
UNIVERSITY OF ILLINOIS
UNIVERSITY OF CALIFORNIA, BERKELEY

Wiley Classics Library Edition Published 1988



INTERSCIENCE PUBLISHERS
A DIVISION OF JOHN WILEY & SONS
New York • Chichester • Brisbane • Toronto

Contents

PART II. SPECTRAL THEORY

Self Adjoint Operators in Hilbert Space

IX. <i>B</i> -Algebras	859
1. Preliminary Notions	859
2. Commutative <i>B</i> -Algebras	868
3. Commutative <i>B</i> *-Algebras	874
4. Exercises	879
5. Notes and Remarks	883
X. Bounded Normal Operators in Hilbert Space	887
1. Terminology and Preliminary Notions	887
2. The Spectral Theorem for Bounded Normal Operators	895
3. Eigenvalues and Eigenvectors	902
4. Unitary, Self Adjoint, and Positive Operators	905
5. Spectral Representation	909
6. A Formula for the Spectral Resolution	920
7. Perturbation Theory	921
8. Exercises	928
9. Notes and Remarks	926
XI. Miscellaneous Applications	937
1. Compact Groups	937
2. Almost Periodic Functions	945
3. Convolution Algebras	949
4. Closure Theorems	978
5. Exercises	1001
6. Hilbert-Schmidt Operators	1009
7. The Hilbert Transform and the Calderón-Zygmund Inequality	1044
8. Exercises	1073
9. The Classes C_μ of Compact Operators. Generalized Carleman Inequalities	1088
10. Subdiagonalization of Compact Operators	1119
11. Notes and Remarks	1145

XII. Unbounded Operators in Hilbert Space	1185
1. Introduction	1185
2. The Spectral Theorem for Unbounded Self Adjoint Operators	1191
3. Spectral Representation of Unbounded Self Adjoint Transformations	1205
4. The Extensions of a Symmetric Transformation	1222
5. Semi-bounded Symmetric Operators	1240
6. Unitary Semi-groups	1242
7. The Canonical Factorization	1245
8. Moment Theorems	1250
9. Exercises	1257
10. Notes and Remarks	1268
XIII. Ordinary Differential Operators	1278
1. Introduction: Elementary Properties of Formal Differential Operators	1278
2. Adjoints and Boundary Values of Differential Operators	1285
3. Resolvents of Differential Operators	1310
4. Spectral Theory: Compact Resolvents	1330
5. Spectral Theory: General Case	1333
6. Qualitative Theory of the Deficiency Index	1392
7. Qualitative Theory of the Spectrum	1435
8. Examples	1503
9. Exercises	1538
10. Notes and Remarks	1581
XIV. Linear Partial Differential Equations and Operators	1629
1. Introduction. The Cauchy Problem, Local Dependence	1629
2. Notational Conventions and Preliminaries	1635
3. The Theory of Distributions	1644
4. The Theorem of Sobolev	1680
5. Some Geometric Considerations	1699
6. The Elliptic Boundary Value Problem	1703
7. Linear Hyperbolic Equations and the Cauchy Problem	1748
8. Parabolic Equations and Semi-groups	1766
APPENDIX	1773
REFERENCES	1785
NOTATION INDEX	1885
AUTHOR INDEX	1889
SUBJECT INDEX	1899

PART III. SPECTRAL OPERATORS

- XV. Spectral Operators
- XVI. Spectral Operators: Sufficient Conditions
- XVII. Algebras of Spectral Operators
- XVIII. Unbounded Spectral Operators
- XIX. Perturbations of Spectral Operators with Discrete Spectra
- XX. Perturbations of Spectral Operators with Continuous Spectra