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Using the Borsuk-Ulam Theorem

Lectures on Topological Methods
in Combinatorics and Geometry

Written in cooperation
with Anders Björner and Günter M. Ziegler



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