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Local and Semi-Local Bifurcations in Hamiltonian Dynamical Systems

Results and Examples

 Springer

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