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# Open Quantum Systems III

Recent Developments

 Springer

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## Contents

### Topics in Non-Equilibrium Quantum Statistical Mechanics

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