

GEOMETRY
AND THE
IMAGINATION

BY
D. HILBERT
AND
S. COHN-VOSSEN

TRANSLATED BY
P. NEMENYI

CHELSEA PUBLISHING COMPANY
NEW YORK

CONTENTS

PREFACE	iii
---------------	-----

CHAPTER I

THE SIMPLEST CURVES AND SURFACES

§ 1. Plane Curves	1
§ 2. The Cylinder, the Cone, the Conic Sections and Their Surfaces of Revolution	7
§ 3. The Second-Order Surfaces	12
§ 4. The Thread Construction of the Ellipsoid, and Confocal Quadrics	19

APPENDICES TO CHAPTER I

1. The Pedal-Point Construction of the Conics.....	25
2. The Directrices of the Conics.....	27
3. The Movable Rod Model of the Hyperboloid.....	29

CHAPTER II

REGULAR SYSTEMS OF POINTS

§ 5. Plane Lattices	32
§ 6. Plane Lattices in the Theory of Numbers.....	37
§ 7. Lattices in Three and More than Three Dimensions.....	44
§ 8. Crystals as Regular Systems of Points.....	52
§ 9. Regular Systems of Points and Discontinuous Groups of Motions	56
§ 10. Plane Motions and their Composition; Classification of the Discontinuous Groups of Motions in the Plane.....	59
§ 11. The Discontinuous Groups of Plane Motions with Infinite Unit Cells	64
§ 12. The Crystallographic Groups of Motions in the Plane. Regular Systems of Points and Pointers. Division of the Plane into Congruent Cells	70
§ 13. Crystallographic Classes and Groups of Motions in Space. Groups and Systems of Points with Bilateral Symmetry	81
§ 14. The Regular Polyhedra	89

CHAPTER III

PROJECTIVE CONFIGURATIONS

§ 15. Preliminary Remarks about Plane Configurations.....	95
§ 16. The Configurations (7_3) and (8_3)	98
§ 17. The Configurations (9_3)	102
§ 18. Perspective, Ideal Elements, and the Principle of Duality in the Plane	112
§ 19. Ideal Elements and the Principle of Duality in Space. Desargues' Theorem and the Desargues Configuration (10_3)	119
§ 20. Comparison of Pascal's and Desargues Theorems.....	128
§ 21. Preliminary Remarks on Configurations in Space.....	133
§ 22. Reye's Configuration	134
§ 23. Regular Polyhedra in Three and Four Dimensions, and their Projections	143
§ 24. Enumerative Methods of Geometry.....	157
§ 25. Schläfli's Double-Six	164

CHAPTER IV

DIFFERENTIAL GEOMETRY

§ 26. Plane Curves	172
§ 27. Space Curves	178
§ 28. Curvature of Surfaces. Elliptic, Hyperbolic, and Parabolic Points. Lines of Curvature and Asymptotic Lines. Um- bilical Points, Minimal Surfaces, Monkey Saddles.....	183
§ 29. The Spherical Image and Gaussian Curvature.....	193
§ 30. Developable Surfaces, Ruled Surfaces.....	204
§ 31. The Twisting of Space Curves.....	211
§ 32. Eleven Properties of the Sphere.....	215
§ 33. Bendings Leaving a Surface Invariant.....	232
§ 34. Elliptic Geometry	235
§ 35. Hyperbolic Geometry, and its Relation to Euclidean and to Elliptic Geometry	242
§ 36. Stereographic Projection and Circle-Preserving Trans- formations. Poincaré's Model of the Hyperbolic Plane.....	248

§ 37. Methods of Mapping, Isometric, Area-Preserving, Geodesic, Continuous and Conformal Mappings.....	260
§ 38. Geometrical Function Theory. Riemann's Mapping Theorem. Conformal Mapping in Space.....	263
§ 39. Conformal Mappings of Curved Surfaces. Minimal Surfaces. Plateau's Problem	268

CHAPTER V KINEMATICS

§ 40. Linkages	272
§ 41. Continuous Rigid Motions of Plane Figures.....	275
§ 42. An Instrument for Constructing the Ellipse and its Roulettes	283
§ 43. Continuous Motions in Space	285

CHAPTER VI TOPOLOGY

§ 44. Polyhedra	290
§ 45. Surfaces	295
§ 46. One-Sided Surfaces	302
§ 47. The Projective Plane as a Closed Surface.....	313
§ 49. Topological Mappings of a Surface onto Itself. Fixed Points. Classes of Mappings. The Universal Covering Surface of the Torus.....	324
§ 50. Conformal Mapping of the Torus.....	330
§ 51. The Problem of Contiguous Regions, The Thread Problem, and the Color Problem	333

APPENDICES TO CHAPTER VI

1. The Projective Plane in Four-Dimensional Space.....	340
2. The Euclidean Plane in Four-Dimensional Space.....	341
INDEX	345