TOPOLOGICAL GROUPS

Characters, Dualities, and Minimal Group Topologies

DIKRAN N. DIKRANJAN
LUCHEZAR N. STOYANOV
Bulgarian Academy of Sciences
Sofia, Bulgaria

IVAN R. PRODANOV
Sofia University
Sofia, Bulgaria

MARCEL DEKKER, INC **New York and Basel**

Contents

PREFACE iii

1. EXISTENCE OF CHARACTERS 1

1.1 Characters of Discrete Abelian Groups 1
1.2 Bogoliouboff Lemma 3
1.3 Følner Lemma 7
1.4 Følner Theorem 12
1.5 Notes 17
1.6 Exercises 24

2. APPLICATIONS OF THE FØLNER THEOREM 30

2.1 Topological Groups: Preliminary Facts 30
2.2 Approximation of Almost Periodic Functions 34
2.3 Precompact Group Topologies on Abelian Groups 35
2.4 Haar Integral in Locally Compact Abelian Groups 38
2.5 Minimal Topologies on Abelian Groups 43
2.6 Localization 46
2.7 Precompactness of Minimal Abelian Groups 48
2.8 Submaximal Topologies on Abelian Groups 55
2.9 Notes 57
2.10 Exercises 64

3. PONTRYAGIN DUALITY 74

3.1 Pontryagin Duality in the Compact-Discrete Case 74
3.2 Pontryagin Duality for Locally Compact Abelian Groups 81
3.3 Structure of Locally Compact Abelian Groups 88
3.4 Uniqueness of Pontryagin Duality 96
3.5 The Group of p-adic Integers 106
3.6 The Group $\mathbb{Q}^*$ 113
3.7 Notes 117
3.8 Exercises 120

4. QUASI-TORSION ELEMENTS IN TOPOLOGICAL GROUPS 127

4.1 Quasi-p-Torsion Elements 127
4.2 Compact Abelian Groups that Do Not Contain $\mathbb{Z}_p^{\mathbb{N}}$ 134
4.3 Totally Minimal Topological Groups 142
4.4 Notes 148
4.5 Exercises 153

5. MINIMAL ABELIAN GROUPS 162

5.1 Description of Minimal Topologies on Some Abelian Groups 162
5.2 A Characterization of the Groups of p-adic Integers 170
5.3 Cardinal Invariants of Minimal Abelian Groups 172
5.4 Notes 176
5.5 Exercises 179

6. MINIMALITY OF PRODUCTS OF TOPOLOGICAL ABELIAN GROUPS 183

6.1 Finite Products and Perfectly Minimal Groups 183
6.2 Minimality Criterion for Products 194
6.3 Critical Power of Minimality 201
6.4 Notes 207
6.5 Exercises 208

7. MINIMAL AND TOTALLY MINIMAL NONABELIAN GROUPS 214

7.1 Symmetric Topological Groups 214
7.2 Semidirect Products and Their Applications to Minimal Group Topologies 221
7.3 The Three-Space Problem and Products of (Totally) Minimal Groups 227
7.4 Total Minimality of $SL(2, \mathbb{R})$ 235
7.5 Cardinal Invariants of Minimal Groups 241
7.6 Total Minimality of the Unitary Groups 246
7.7 Notes 259
7.8 Exercises 263

NOTATION 269

REFERENCES 273

INDEX 285