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Improved Bonferroni Inequalities via Abstract Tubes

Inequalities and Identities
of Inclusion-Exclusion Type



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