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Easy as π ?

An Introduction to Higher Mathematics

Translated by Robert G. Burns

With 60 Illustrations



Springer

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