O.A. Ivanov

Easy as π?

An Introduction to Higher Mathematics

Translated by Robert G. Burns

With 60 Illustrations

Springer

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