

Mathematical Methods
and Theory
in Games, Programming,
and Economics

VOLUME I:
MATRIX GAMES, PROGRAMMING, AND
MATHEMATICAL ECONOMICS

VOLUME II:
THE THEORY OF INFINITE GAMES

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TWO VOLUMES BOUND AS ONE

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