

**SOLUTION
TECHNIQUES
FOR
ELEMENTARY
PARTIAL DIFFERENTIAL
EQUATIONS**

CHRISTIAN CONSTANDA

PROFESSOR OF MATHEMATICS
UNIVERSITY OF STRATHCLYDE
GLASGOW, UK

AND

VISITING PROFESSOR OF MATHEMATICS
UNIVERSITY OF TULSA
TULSA, OKLAHOMA, USA



CHAPMAN & HALL/CRC

A CRC Press Company

Boca Raton London New York Washington, D.C.

Contents

Foreword	xi
Preface	xiii
Chapter 1. Ordinary Differential Equations: Brief Revision	1
1.1. First-Order Equations	1
1.2. Homogeneous Linear Equations with Constant Coefficients	3
1.3. Nonhomogeneous Linear Equations with Constant Coefficients	5
1.4. Linear Operators	6
Exercises	8
Chapter 2. Fourier Series	11
2.1. The Full Fourier Series	11
2.2. Fourier Sine Series	17
2.3. Fourier Cosine Series	21
2.4. Convergence and Differentiation	24
Exercises	25
Chapter 3. Sturm-Liouville Problems	29
3.1. Regular Sturm-Liouville Problems	29
3.2. Other Sturm-Liouville Problems	40
Exercises	44
Chapter 4. Three Fundamental Equations of Mathematical Physics	49
4.1. The Heat Equation	49
4.2. The Laplace Equation	57
4.3. The Wave Equation	63
Chapter 5. The Method of Separation of Variables	69
5.1. The Heat Equation	69
5.2. The Wave Equation	82
5.3. The Laplace Equation	88
5.4. Equations with More than Two Variables	96
Exercises	104

Chapter 6. Linear Nonhomogeneous Problems	111
6.1. Equilibrium Solutions	111
6.2. Nonhomogeneous Problems	116
Exercises	120
Chapter 7. The Method of Eigenfunction Expansion	123
7.1. The Heat Equation	123
7.2. The Wave Equation	129
7.3. The Laplace Equation	132
Exercises	135
Chapter 8. The Fourier Transformations	141
8.1. The Full Fourier Transformation	141
8.2. The Fourier Sine and Cosine Transformations	147
Exercises	154
Chapter 9. The Laplace Transformation	157
9.1. Definition and Properties	157
9.2. Applications	162
Exercises	172
Chapter 10. The Method of Green's Functions	175
10.1. The Heat Equation	175
10.2. The Laplace Equation	182
10.3. The Wave Equation	185
Exercises	191
Chapter 11. General Second-Order Linear Partial Differential Equations with Two Independent Variables	195
11.1. The Canonical Form	195
11.2. Hyperbolic Equations	199
11.3. Parabolic Equations	202
11.4. Elliptic Equations	204
Exercises	205

Chapter 12. The Method of Characteristics	207
12.1. First-Order Linear Equations	207
12.2. First-Order Quasilinear Equations	214
12.3. The One-Dimensional Wave Equation	215
Exercises	222
Chapter 13. Perturbation and Asymptotic Methods	225
13.1. Asymptotic Series	225
13.2. Regular Perturbation Problems	228
13.3. Singular Perturbation Problems	234
Exercises	240
Appendix	243
Bibliography	247
Index	249