

Models of Peano Arithmetic

RICHARD KAYE

*Jesus College
Oxford*

CLARENDON PRESS · OXFORD

1991

Contents

0. Background	1
0.1 Background model theory	2
0.2 Background recursion theory	5
1. The standard model	10
2. Discretely ordered rings	16
2.1 The axioms of PA^*	16
2.2 Initial segments and end-extensions	21
3. Gödel incompleteness	28
3.1 Σ_1 formulas and the computable sets	28
3.2 Incompleteness	35
4. The axioms of Peano arithmetic	42
4.1 Alternative induction schemes	43
4.2 Introducing new relations and functions	47
5. Some number theory in PA	53
5.1 Euclidean division and Bézout's theorem	53
5.2 Gödel's lemma	58
5.3 The infinitude of primes	65
6. Models of PA	69
6.1 Overspill	70
6.2 The order-type of a model of PA	73
7. Collection	78
7.1 The arithmetic hierarchy	78
7.2 Cofinal extensions	86
8. Prime models	91
8.1 Definable elements	91
8.2 End-extensions	96

9. Satisfaction	104
9.1 Sequences	105
9.2 Syntax	108
9.3 Semantics	119
10. Subsystems of <i>PA</i>	130
10.1 Σ_n -definable elements	130
10.2 Σ_n -elementary initial segments	134
11. Saturation	141
11.1 Coded sets	141
11.2 Σ_n -recursive saturation	146
11.3 Tennenbaum's theorem	153
12. Initial segments	159
12.1 Friedman's theorem	160
12.2 Variations	166
13. The standard system	172
13.1 Scott sets	172
13.2 The arithmetized completeness theorem	186
14. Indicators	193
14.1 What are indicators?	193
14.2 Recursively axiomatized theories have indicators	198
14.3 The Paris–Harrington theorem	206
15. Recursive saturation	223
15.1 Satisfaction classes	224
15.2 Resplendency	247
16. Suggestions for further reading	266
Bibliography	276
Index	285