Theoria Combinationis Observationum Erroribus Minimis Obnoxiae

Pars Prior ◆ *Pars Posterior* ◆ *Supplementum*

By Carl Friedrich Gauss

Theory of the Combination of Observations Least Subject to Errors

Part One ◆ *Part Two* ◆ *Supplement*

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