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# Partial Differential Equations

Analytical Solution Techniques

Second Edition

With 128 Illustrations



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