



An Introduction to Variational Inequalities and Their Applications





David Kinderlehrer

Guido Stampacchia

siam

Society for Industrial and Applied Mathematics
Philadelphia

Contents

<i>Preface to the Classics Edition</i>	xiii
<i>Preface</i>	xvii
<i>Glossary of Notations</i>	xix
Introduction	1
Chapter I Variational Inequalities in \mathbb{R}^n	
1. Fixed Points	7
2. The Characterization of the Projection onto a Convex Set	8
3. A First Theorem about Variational Inequalities	11
4. Variational Inequalities	13
5. Some Problems Which Lead to Variational Inequalities	15
Comments and Bibliographical Notes	18
Exercises	18
Chapter II Variational Inequalities in Hilbert Space	
1. Bilinear Forms	23
2. Existence of a Solution	24
3. Truncation	27
4. Sobolev Spaces and Boundary Value Problems	28
5. The Weak Maximum Principle	35
6. The Obstacle Problem: First Properties	40
7. The Obstacle Problem in the One Dimensional Case	47
Appendix A. Sobolev Spaces	49
Appendix B. Solutions to Equations with Bounded Measurable Coefficients	62

Appendix C. Local Estimates of Solutions	66
Appendix D. Hölder Continuity of the Solutions	72
Comments and Bibliographical Notes	76
Exercises	77
 Chapter III Variational Inequalities for Monotone Operators	
1. An Abstract Existence Theorem	83
2. Noncoercive Operators	87
3. Semilinear Equations	93
4. Quasi-Linear Operators	94
Comments and Bibliographical Notes	100
Exercises	101
 Chapter IV Problems of Regularity	
1. Penalization	105
2. Dirichlet Integral	106
3. Coercive Vector Fields	113
4. Locally Coercive Vector Fields	116
5. Another Penalization	120
6. Limitation of Second Derivatives	124
7. Bounded Variation of Au	130
8. Lipschitz Obstacles	134
9. A Variational Inequality with Mixed Boundary Conditions	139
Appendix A. Proof of Theorem 3.3	143
Comments and Bibliographical Notes	146
Exercises	147
 Chapter V Free Boundary Problems and the Coincidence Set of the Solution	
1. Introduction	149
2. The Hodograph and Legendre Transformations	153
3. The Free Boundary in Two Dimensions	155
4. A Remark about Singularities	166
5. The Obstacle Problem for a Minimal Surface	167
6. The Topology of the Coincidence Set When the Obstacle Is Concave	173
7. A Remark about the Coincidence Set in Higher Dimensions	178
Comments and Bibliographical Notes	181
Exercises	182
 Chapter VI Free Boundary Problems Governed by Elliptic Equations and Systems	
1. Introduction	184
2. Hodograph and Legendre Transforms: The Theory of a Single Equation	185

3. Elliptic Systems	190
4. A Reflection Problem	202
5. Elliptic Equations Sharing Cauchy Data	204
6. A Problem of Two Membranes	212
Comments and Bibliographical Notes	218
Exercises	218

Chapter VII Applications of Variational Inequalities

1. Introduction	222
2. A Problem in the Theory of Lubrication	223
3. The Filtration of a Liquid through a Porous Medium	227
4. The Resolution of the Filtration Problem by Variational Inequalities	235
5. The Filtration of a Liquid through a Porous Medium with Variable Cross Section	242
6. The Resolution of the Filtration Problem in Three Dimensions	249
7. Flow past a Given Profile: The Problem in the Physical Plane	257
8. Flow past a Given Profile: Resolution by Variational Inequalities	260
9. The Deflection of a Simply Supported Beam	270
Comments and Bibliographical Notes	273
Exercises	274

Chapter VIII A One Phase Stefan Problem

1. Introduction	278
2. Existence and Uniqueness of the Solution	281
3. Smoothness Properties of the Solution	289
4. The Legendre Transform	297
Comments and Bibliographical Notes	299

Bibliography	300
---------------------	-----

<i>Index</i>	309
--------------	-----