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# Geometric Methods in the Algebraic Theory of Quadratic Forms

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### Cohomologie non ramifiée des quadriques

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