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Mathematical Theory of Nonequilibrium Steady States

On the Frontier
of Probability
and Dynamical Systems



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