

Nonlinear time series analysis

Holger Kantz

Max Planck Institute for Physics of
Complex Systems, Dresden

Thomas Schreiber

Physics Department, University of Wuppertal



CAMBRIDGE
UNIVERSITY PRESS

Contents

Preface	xii
Acknowledgements	xv
Part 1 Basic topics 1	
Chapter 1	Introduction: Why nonlinear methods? 3
Chapter 2	Linear tools and general considerations 13
2.1	Stationarity and sampling 13
2.2	Testing for stationarity 15
2.3	Linear correlations and the power spectrum 18
2.3.1	<i>Stationarity and the low-frequency component in the power spectrum</i> 22
2.4	Linear filters 23
2.5	Linear predictions 25
Chapter 3	Phase space methods 29
3.1	Determinism: Uniqueness in phase space 29
3.2	Delay reconstruction 33
3.3	Finding a good embedding 34
3.4	Visual inspection of data 37
3.5	Poincaré surface of section 37

Chapter 4 Determinism and predictability	42
4.1 Sources of predictability	43
4.2 Simple nonlinear prediction algorithm	44
4.3 Verification of successful prediction	46
4.4 Probing stationarity with nonlinear predictions	49
4.5 Simple nonlinear noise reduction	51
Chapter 5 Instability: Lyapunov exponents	58
5.1 Sensitive dependence on initial conditions	58
5.2 Exponential divergence	59
5.3 Measuring the maximal exponent from data	62
Chapter 6 Self-similarity: Dimensions	69
6.1 Attractor geometry and fractals	69
6.2 Correlation dimension	70
6.3 Correlation sum from a time series	72
6.4 Interpretation and pitfalls	75
6.5 Temporal correlations, nonstationarity, and space time separation plots	81
6.6 Practical considerations	84
6.7 A useful application: Determination of the noise level	86
Chapter 7 Using nonlinear methods when determinism is weak	97
7.1 Testing for nonlinearity with surrogate data	93
7.1.1 <i>The null hypothesis</i>	95
7.1.2 <i>How to make surrogate data sets</i>	96
7.1.3 <i>Which statistics to use</i>	99
7.1.4 <i>What can go wrong</i>	102
7.1.5 <i>What we have learned</i>	103
7.2 Nonlinear statistics for system discrimination	104
7.3 Extracting qualitative information from a time series	108
Chapter 8 Selected nonlinear phenomena	112
8.1 Coexistence of attractors	112
8.2 Transients	113
8.3 Intermittency	114

8.4	Structural stability	118
8.5	Bifurcations	119
8.6	Quasi-periodicity	121
Part 2 Advanced topics 123		
Chapter 9	Advanced embedding methods	125
9.1	Embedding theorems	125
9.1.1	<i>Whitney's embedding theorem</i>	126
9.1.2	<i>Takens's delay embedding theorem</i>	127
9.2	The time lag	130
9.3	Filtered delay embeddings	134
9.3.1	<i>Derivative coordinates</i>	134
9.3.2	<i>Principal component analysis</i>	135
9.4	Fluctuating time intervals	139
9.5	Multichannel measurements	141
9.5.1	<i>Equivalent variables at different positions</i>	141
9.5.2	<i>Variables with different physical meanings</i>	142
9.5.3	<i>Distributed systems</i>	143
9.6	Embedding of interspike intervals	145
Chapter 10	Chaotic data and noise	150
10.1	Measurement noise and dynamical noise	150
10.2	Effects of noise	151
10.3	Nonlinear noise reduction	154
10.3.1	<i>Noise reduction by gradient descent</i>	155
10.3.2	<i>Local projective noise reduction</i>	156
10.3.3	<i>Implementation of locally projective noise reduction</i>	159
10.3.4	<i>How much noise is taken out?</i>	163
10.3.5	<i>Consistency tests</i>	167
10.4	An application: Foetal ECG extraction	168
Chapter 11	More about invariant quantities	172
11.1	Ergodicity and strange attractors	173
11.2	Lyapunov exponents II	174
11.2.1	<i>The spectrum of Lyapunov exponents and invariant manifolds</i>	174

11.2.2	<i>Flows versus maps</i>	176
11.2.3	<i>Tangent space method</i>	177
11.2.4	<i>Spurious exponents</i>	178
11.2.5	<i>Almost two-dimensional flows</i>	184
11.3	<i>Dimensions II</i>	184
11.3.1	<i>Generalised dimensions, multifractals</i>	186
11.3.2	<i>Information dimension from a time series</i>	188
11.4	<i>Entropies</i>	189
11.4.1	<i>Chaos and the flow of information</i>	189
11.4.2	<i>Entropies of a static distribution</i>	191
11.4.3	<i>The Kolmogorov–Sinai entropy</i>	193
11.4.4	<i>Entropies from time series data</i>	194
11.5	<i>How things are related</i>	198
11.5.1	<i>Pesin's identity</i>	198
11.5.2	<i>Kaplan–Yorke conjecture</i>	199
Chapter 12	Modelling and forecasting	202
12.1	<i>Stochastic models</i>	204
12.1.1	<i>Linear filters</i>	204
12.1.2	<i>Nonlinear filters</i>	206
12.1.3	<i>Markov models</i>	207
12.2	<i>Deterministic dynamics</i>	207
12.3	<i>Local methods in phase space</i>	208
12.3.1	<i>Almost model free methods</i>	209
12.3.2	<i>Local linear fits</i>	209
12.4	<i>Global nonlinear models</i>	211
12.4.1	<i>Polynomials</i>	211
12.4.2	<i>Radial basis functions</i>	212
12.4.3	<i>Neural networks</i>	213
12.4.4	<i>What to do in practice</i>	214
12.5	<i>Improved cost functions</i>	215
12.5.1	<i>Overfitting and model costs</i>	216
12.5.2	<i>The errors-in-variables problem</i>	217
12.6	<i>Model verification</i>	219
Chapter 13	Chaos control	223
13.1	<i>Unstable periodic orbits and their invariant manifolds</i>	224
13.1.1	<i>Locating periodic orbits</i>	225

13.1.2	<i>Stable/unstable manifolds from data</i>	229
13.2	OGY-control and derivates	231
13.3	Variants of OGY-control	234
13.4	Delayed feedback	235
13.5	Chaos suppression without feedback	235
13.6	Tracking	236
13.7	Related aspects	237
Chapter 14	Other selected topics	239
14.1	High dimensional chaos	239
14.1.1	<i>Analysis of higher dimensional signals</i>	241
14.1.2	<i>Spatially extended systems</i>	245
14.2	Analysis of spatiotemporal patterns	247
14.3	Multiscale or self-similar signals, wavelets	249
14.3.1	<i>Dynamical origin of multiscale signals</i>	250
14.3.2	<i>Scaling laws</i>	252
14.3.3	<i>Wavelet analysis</i>	254
Appendix A	Efficient neighbour searching	257
Appendix B	Program listings	262
Appendix C	Description of the experimental data sets	278
References		288
Index		300