

Giovanni Gallavotti

Foundations of Fluid Dynamics

With 41 Figures and 400 Problems



Springer

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Table of Contents

1	Continua and Generalities About Their Equations	1
1.1	Continua	1
1.1.1	Mass Conservation	2
1.1.2	Momentum Conservation (First Cardinal Equation)	3
1.1.3	Angular Momentum Conservation	3
1.1.4	Energy Conservation	5
1.1.5	Law of Thermodynamics and Entropy Balance	6
	Problems	8
1.2	Equations of Motion of a Fluid in General. Ideal and Incompressible Cases. Incompressible Euler, Navier–Stokes and Navier–Stokes–Fourier Equations	15
1.2.1	Incompressible Non-Viscous Fluid (Euler Equations)	16
1.2.2	Incompressible, Non-Heat-Conducting, Viscous Fluid (Navier–Stokes Equations)	17
1.2.3	Incompressible Thermoconducting Viscous Fluid	17
1.2.4	On the Physical Meaning of an Incompressibility Condition	18
1.2.5	The Case of Incompressible Euler Equations	19
1.2.6	The Case of the Navier–Stokes Equations	20
1.2.7	The Case in Which Heating in Adiabatic Compressions is not Negligible	20
	Problems. Stokes Formula	22
1.3	The Rescaling Method and Estimates of the Approximations	24
1.3.1	The Incompressible Euler Equation	24
1.3.2	The Incompressible Navier–Stokes Equation	29
1.4	Elements of Hydrostatics	31
1.4.1	Hydrostatics in the Absence of Thermoconduction	31
1.4.2	Hydrostatics in the Presence of Thermoconduction	35
1.4.3	Current Lines and the Bernoulli Theorem	37
	Problems	39
1.5	The Convection Problem. Rayleigh’s Equations	43
1.5.1	General Considerations on Convection	43

1.5.2	The Physical Assumptions of Rayleigh's Convection Model	44
1.5.3	The Rayleigh Model	47
1.5.4	Rescalings: A Systematic Analysis	51
	Problem	53
1.6	Kinematics: Incompressible Fields, Vector Potentials, Decompositions of a General Field	53
1.6.1	Incompressible Fields in the Whole Space as Rotations of a Vector Potential	53
1.6.2	An Incompressible Field in a Finite Convex Volume Ω as a Rotation: Some Sufficient Conditions	54
1.6.3	Ambiguities for Vector Potentials of Incompressible Fields	57
1.6.4	A Regular Vector Field in Ω can be Represented as the Sum of a Rotation Field and a Gradient Field	57
1.6.5	The Space $X_{\text{rot}}(\Omega)$ and Its Complement in $L_2(\Omega)$...	58
1.6.6	The "Gradient-Solenoid" Decomposition of a Regular Vector Field	60
	Problems and Extensions	60
1.7	Vorticity Conservation in the Euler Equation. Clebsch Potentials and Hamiltonian Form of Euler Equations. Bidimensional Fluids	66
1.7.1	Thomson's Theorem	66
1.7.2	Irrotational Isentropic Flows	67
1.7.3	Eulerian Vorticity Equation in General and in Bidimensional Flows	68
1.7.4	Hamiltonian Form of the Incompressible Euler Equations and Clebsch Potentials. Normal Velocity Fields	69
1.7.5	Lagrangian Form of the Incompressible Euler Equations. Hamiltonian Form on the Group of Diffeomorphisms	73
1.7.6	Hamiltonian Form of the Eulerian Potential Flows; Example in Two Dimensions	75
1.7.7	Small Waves	78
	Problems. Sound and Surface Waves. Radiated Energy	78
2	Empirical Algorithms	83
2.1	Incompressible Euler and Navier-Stokes Fluid Dynamics. First Empirical Solutions Algorithms. Auxiliary Friction and Heat Equation Comparison Methods	83
2.1.1	Euler Equation	83
2.1.2	Navier-Stokes-Euler Equation	85
2.1.3	Navier-Stokes Equations and Algorithmic Difficulties	85

2.1.4	Heat Equation	87
2.1.5	An Empirical Algorithm for NS	89
2.1.6	Precision of the Same Algorithms Applied to the Heat Equation (Q1)	91
2.1.7	Analysis of the Precision of the Algorithms in the Heat Equation Case Q2	92
2.1.8	Comments	94
	Problems. Well-Posedness of the Heat Equation and Other Remarks	96
2.2	Another Class of Empirical Algorithms. Spectral Method. Stokes Problem. Gyroscopic Analogy	99
2.2.1	Periodic Boundary Conditions: Spectral Algorithm and “Reduction” to an Ordinary Differential Equation	99
2.2.2	Spectral Method in a Domain Ω with Boundary and the Boundary Conditions Problem	102
2.2.3	The Stokes Problem	104
2.2.4	Comments	105
2.2.5	Gyroscopic Analogy in $d = 2$	106
2.2.6	Gyroscopic Analogy in $d = 3$	107
	Problems. Interior and Boundary Regularity of Solutions of Elliptic Equations and for the Stokes Equation	110
2.3	Vorticity Algorithms for Incompressible Euler and Navier–Stokes Fluids. The $d = 2$ Case	120
	Problems. Few Vortices Hamiltonian Motions. Periodic Green Function	126
2.4	Vorticity Algorithms for Incompressible Euler and Navier–Stokes Fluids. The $d = 3$ Case	128
2.4.1	Regular Filaments. Divergences and Infinities	129
2.4.2	Thin Filament. Smoke Ring	131
2.4.3	Irregular Filaments: Brownian Filaments	134
2.4.4	Irregular Filaments: Quasi-Periodic Filaments	138
	Problems	140
3	Analytical Theories and Mathematical Aspects	143
3.1	Spectral Method and Local Existence, Regularity and Uniqueness Theorems for Euler and Navier–Stokes Equations, $d \geq 2$	143
	Problems. Classical Local Theory for the Euler and Navier–Stokes Equations with Periodic Conditions	152
3.2	Weak Global Existence Theorems for NS. Autoregularization, Existence, Regularity and Uniqueness for $d = 2$	159
	Problems	176

3.3	Regularity: Partial Results for the NS Equation in $d = 3$.	
	The Theory of Leray	176
3.3.1	Leray's Regularization	177
3.3.2	Properties of the Regularized Equation and New Weak Solutions	178
3.3.3	The Local Bounds of Leray. Uniformity in the Regularization Parameter	180
3.3.4	Local Existence and Regularity. Leray's Local Theorem	185
3.3.5	Exceptionality of Singularities. Global Leray's Theorem. Leray's Solutions	187
3.3.6	Characterization of the Singularities. The Leray-Serrin Theorem	188
3.3.7	Vorticity Orientation Uncertainty at Singularities ...	190
3.3.8	Large Containers	190
	Problems. Further Results in Leray's Theory	192
3.4	Fractal Dimension of Singularities of the Navier-Stokes Equation, $d = 3$	197
3.4.1	Dimension and Measure of Hausdorff	197
3.4.2	Hausdorff Dimension of Singular Times in the Navier-Stokes Solutions ($d = 3$)	199
3.4.3	Hausdorff Dimension in Space-Time of the Solutions of NS, ($d = 3$)	200
	Problems	204
3.5	Local Homogeneity and Regularity. CKN Theory	205
3.5.1	Energy Balance for Weak Solutions	205
3.5.2	General Sobolev Inequalities and Further a Priori Bounds	207
3.5.3	Pseudo-Navier-Stokes Velocity-Pressure Pairs. Scaling Operators	209
3.5.4	The Theorems of Scheffer and of Caffarelli-Kohn-Nirenberg	212
3.5.5	Proof that the Renormalization Map Contracts	214
	Problems. The CKN Theory	217
4	Incipient Turbulence and Chaos	227
4.1	Fluid Theory in the Absence of Existence and Uniqueness Theorems for the Basic Fluid Dynamics Equations. Truncated NS Equations. (The Rayleigh and Lorenz Models)	227
4.1.1	The Two-Dimensional Saltzman Equations	230
4.1.2	A Priori Estimates	231
4.1.3	The Lorenz Model	232
4.1.4	Truncated NS Models	235
	Problems	237

4.2	Onset of Chaos. Elements of Bifurcation Theory	239
4.2.1	Laminar Motion	245
4.2.2	Loss of Stability of the Laminar Motion	246
4.2.3	The Notion of Genericity	247
4.2.4	Generic Routes to the Loss of Stability of Laminar Motion. Spontaneously Broken Symmetry	253
	Problems. Hopf Bifurcation	256
4.3	Bifurcation Theory. End of the Onset of Turbulence	259
4.3.1	Stability Loss of Periodic Motion. Hopf Bifurcation ..	259
4.3.2	Loss of Stability of Periodic Motion. Period Doubling Bifurcation	263
4.3.3	Stability Loss of Periodic Motion. Bifurcation Through $\lambda = 1$	264
4.3.4	And Then? Chaos! and Its Scenarios	265
4.3.5	Conclusions	270
4.3.6	The Experiments	272
	Problems	272
4.4	Dynamical Tables	275
4.4.1	Bifurcations and Their Graphical Representation ...	277
4.4.2	An Example: The Dynamical Table for the Model NS_5	279
4.4.3	The Lorenz Model Table	282
4.4.4	Remarks	283
4.4.5	Another Example: The Dynamic Table of the NS_7 Model	284
4.4.6	Table of the Two-Dimensional Navier–Stokes Equation at Small Reynolds Number	285
	Problems	286
5	Ordering Chaos	287
5.1	Quantitative Description of Chaotic Motions	
	Before Developed Turbulence. Continuous Spectrum	287
5.1.1	Diophantine Quasi-Periodic Spectra	288
5.1.2	Continuous Spectrum	290
5.1.3	An Example: Non-Euclidean Geometry	292
5.1.4	A Further Example: The Billiard Ball	294
	Problems:	
	Ergodic Theory of Motions on Surfaces of Constant Non-Positive Curvature	295
	Ergodic Theory of Quasi-Periodic Motion, i.e. of Geodesic Motion on a Torus	296
	Ergodic Theory of Geodesic Flows on Surfaces of Constant Negative Curvature	302

5.2	Timed Observations. Random Data	307
	Problems	316
5.3	Dynamical Systems Types. Statistics on Attracting Sets	319
	Problems	327
5.4	Dynamical Bases and Lyapunov Exponents	330
	Philosophical Problems	342
	Further Problems. Theorems of <i>Oseledec</i> , <i>Raghunathan</i> , and <i>Ruelle</i>	345
5.5	SRB Statistics. Attractors and Attracting Sets.	
	Fractal Dimension	351
5.5.1	“Physical” (i.e. SRB) Probability Distributions	351
5.5.2	Structure of Axiom A Attractors. Heuristic Considerations	355
5.5.3	Attractive Sets and Attractors. Fractal Dimensions	362
	Problems	365
5.6	Ordering of Chaos. Entropy and Complexity	367
5.6.1	Complete Observations and Formal Symbolic Dynamics	367
5.6.2	Complexity of Sequences of Symbols. Shannon–McMillan Theorem	372
5.6.3	Entropy of a Dynamical System and the Kolmogorov–Sinai Theory	375
	Problems	378
5.7	Symbolic Dynamics. Lorenz Model. Ruelle’s Principle	380
5.7.1	Expansive Maps on $[0, 1]$. Infinity of the Number of Invariant Distributions	380
5.7.2	Application to the Lorenz Model	384
5.7.3	Hyperbolic Maps and Markovian Pavements	385
5.7.4	Arnold Cat Map as a Paradigm for the Properties of Markov Pavements	389
5.7.5	More General Hyperbolic Maps and Their Markov Pavements	392
5.7.6	Representation of the SRB Distribution via Markovian Pavements	394
	Problems	396
6	Developed Turbulence	403
6.1	Functional Integral Representation of Stationary Distributions	403
	Problems	410
6.2	Phenomenology of Developed Turbulence and the Kolmogorov Laws	412
6.2.1	Energy Dissipation: Inertial and Viscous Scales	412
6.2.2	Digression on the Physical Meaning of “ $\nu \rightarrow 0$ ”	413
6.2.3	The K41 Tridimensional Theory	414

6.2.4	Bidimensional Theory	420
6.2.5	Remarks on the K41 Theory	422
6.2.6	The Dissipative Euler Equation	424
6.2.7	The Ruelle–Lieb Bounds and K41 Theory	426
	Problems. Dissipation and Attractor Dimension as $\nu \rightarrow 0$.	
	Kolmogorov’s Skew Correlations	427
6.3	The Shell Model. Multifractal Statistics	431
	Problems	439
7	Statistical Properties of Turbulence	441
7.1	Viscosity, Reversibility, and Irreversible Dissipation	441
7.1.1	Reversible Equations for Dissipative Fluids	441
7.1.2	Microscopic Reversibility and Macroscopic Irreversibility	446
7.1.3	Attractors and Attractive Sets	449
	Problems	451
7.2	Reversibility, Axiom C, Chaotic Hypothesis	452
7.2.1	The SRB Distribution, and Other Invariant Distributions	452
7.2.2	Attractors and Reversibility. Unbreakability of Time Reversal Symmetry	454
7.2.3	An Example	455
7.2.4	The Axiom C	457
7.2.5	The Chaotic Hypothesis	461
7.3	Chaotic Hypothesis, Fluctuation Theorem and Onsager Reciprocity. Entropy Driven Intermittency	463
7.3.1	The Fluctuation Theorem	463
7.3.2	Onsager’s Reciprocity and the Chaotic Hypothesis	469
7.3.3	A Fluid-Dynamic Application	471
7.3.4	Physical Interpretation of the Fluctuation Relations. Onsager–Machlup Fluctuations. Entropy-Driven Intermittency	472
	Appendix. Onsager Reciprocity as a Consequence of the Fluctuation Theorem	474
7.4	The Structure of the Attractor for the Navier–Stokes Equations. Dissipative Euler Equations. Barometric Formula	477
7.4.1	Reversible and Irreversible Equations for a Real Fluid	477
7.4.2	Axiom C and the Pairing Rule	482
7.4.3	Relation Between the NS and ED Equations: The Barometric Formula	488
	Problem	490

XVIII Table of Contents

Bibliography	491
Name Index	501
Subject Index	503
Citations Index	511