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# Convex Bodies and Algebraic Geometry

An Introduction to the  
Theory of Toric Varieties

With 42 Figures



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