Matroid Theory

JAMES G. OXLEY

Department of Mathematics Louisiana State University

OXFORD NEW YORK TOKYO OXFORD UNIVERSITY PRESS

Contents

Prelimin	aries	1
Chapter	1. Basic definitions and examples	7
1.	Independent sets and circuits	7
2.	Bases	16
3.	Rank	22
4.	Closure	28
5.	Geometric representations of matroids of small rank	36
6.	Transversal matroids	46
7.	The lattice of flats	51
8.	The greedy algorithm	61
Chapter	2. Duality	68
1.	The definition and basic properties	68
2.	Duals of representable matroids	80
3.	Duals of graphic matroids	89
4.	Duals of transversal matroids	95
Chapter	3. Minors	104
1.	Contraction	104
2.	Minors of certain classes of matroids	111
3.	Projection, flats, and the Scum Theorem	117
Chapte	4. Connectivity	123
1.	Connectivity for graphs and matroids	123
2.	Properties of matroid connectivity	128
3.	More properties of connectivity	133
Chapte	r 5. Graphic matroids	138
1.	Representability	138
2.	Duality in graphic matroids	143
3.	Whitney's 2-Isomorphism Theorem	147
4.	Series-parallel networks	154

x Contents

Chapter	6. Representable matroids	163
1.	Projective geometries	16
2.	Affine geometries	178
3.	Different matroid representations	185
4.	Constructing representations for matroids	190
5.	Representability over finite fields	203
6.	Regular matroids	209
7.	Algebraic matroids	213
8.	Characteristic sets	223
9.	Modularity	230
Chapter	7. Constructions	238
1.	Series and parallel connection	238
2.	Single-element extensions	251
3.	Quotients and related operations	259
Chapter	8. Higher connectivity	271
1.	Tutte's definition	271
2.	Matroid versus graph connectivity	277
3.	3-connected matroids and 2-sums	287
4.	More properties of 3-connected matroids	292
Chapter	9. Binary matroids	300
1.	Characterizations	303
2.	Circuit and cocircuit spaces	310
3.	Other special properties	313
Chapter	10. Ternary matroids	323
1.	Unique representability	323
2.	Some connectivity results	331
3.	The excluded-minor characterization	340
Chapter	11. The Splitter Theorem	346
1.	The theorem and its proof	346
2.	Applications of the Splitter Theorem	356
3.	Variations on the Splitter Theorem	370
Chapter	12. Submodular functions and matroid union	379
1.	Submodular functions	379
2.	The theorems of Hall and Rado	387
3.	Matroid union and its applications	402
4.	Amalgams and the generalized parallel connection	411

	Contents	×
Chapter	13. Regular matroids	42
1.	Proof of the excluded-minor theorem	427
2.	Seymour's decomposition theorem	435
3.	The excluded-minor characterization of graphic matroids	440
4.	Further properties of regular and graphic matroids	45
Chapter	14. Unsolved problems	463
1.	Representability: linear and algebraic	463
2.	Unimodal conjectures	46
3.	The critical problem	468
4.	From graphs to matroids	469
5.	Enumeration	473
6.	Matroid union	47
7.	Gammoids	473
8.	A miscellany	47
Referen	References	
Appendi	Appendix. Some interesting matroids	
Notation		52
Index		52
	n	