

Hervé Pajot

Analytic Capacity,  
Rectifiability,  
Menger Curvature and  
the Cauchy Integral



Springer

## Contents

Introduction	v
Notations and conventions	ix
Chapter 1. Some geometric measure theory	1
1. Carleson measures	1
2. Lipschitz maps	2
3. Hausdorff dimension and Hausdorff measures	4
4. Density properties of Hausdorff measures	7
5. Rectifiable and purely unrectifiable sets	12
Chapter 2. P. Jones' traveling salesman theorem	17
1. The $\beta$ numbers	17
2. Characterization of subsets of rectifiable curves	20
3. Uniformly rectifiable sets	23
Chapter 3. Menger curvature	29
1. Definition and basic properties	29
2. Menger curvature and Lipschitz graphs	31
3. Menger curvature and $\beta$ numbers	32
4. Menger curvature and Cantor type sets	44
5. P. Jones' construction of "good" measures supported on continua	46
Chapter 4. The Cauchy singular integral operator on Ahlfors regular sets	55
1. The Hilbert transform	55
2. Singular integral operators	56
3. The Hardy-Littlewood maximal operator	58
4. The Calderón-Zygmund theory	59
5. The $T1$ and the $Tb$ theorems	59
6. $L^2$ boundedness of the Cauchy singular operator on Lipschitz graphs	61
7. Cauchy singular operator and rectifiability	63
Chapter 5. Analytic capacity and the Painlevé problem	67
1. Removable singularities	67
2. The Painlevé Problem	68
3. Some examples	70
4. Analytic capacity and metric size of sets	74
5. Garnett-Ivanov's counterexample	75
6. Who was Painlevé ?	78
Chapter 6. The Denjoy and Vitushkin conjectures	81
1. The statements	81
2. The standard duality argument	82
3. Proof of the Denjoy conjecture	84

4. Proof of the Vitushkin conjecture	90
5. The Vitushkin conjecture for sets with infinite length	100
Chapter 7. The capacity $\gamma_+$ and the Painlevé Problem	105
1. Melnikov's inequality	105
2. Tolsa's solution of the Painlevé problem	108
3. Concluding remarks and open problems	112
Bibliography	115
Index	119