

Armin Bunde Shlomo Havlin (Eds.)

Fractals and Disordered Systems

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With 165 Figures and 10 Color Plates



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By H. Eugene Stanley (With 30 Figures)

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by Rudolf H. Riedi and Benoit B. Mandelbrot (With 10 Figures)

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