

ESSAYS

ON THE

# THEORY OF NUMBERS

- I. CONTINUITY AND IRRATIONAL NUMBERS
- II. THE NATURE AND MEANING OF NUMBERS

BY

RICHARD DEDEKIND

AUTHORIZED TRANSLATION BY

WOOSTER WOODRUFF BEMAN  
LATE PROFESSOR OF MATHEMATICS  
THE UNIVERSITY OF MICHIGAN

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