

Aldo de Luca • Stefano Varricchio

Finiteness and Regularity in Semigroups and Formal Languages



Springer

Contents

Preface	V
1. Combinatorics on Words	1
1.1 Preliminaries	1
1.2 Infinite words	4
1.3 Metric and topology	7
1.4 Periodicity and conjugacy	9
1.5 Lyndon words	16
1.6 Factorial languages and subword complexity	21
2. Unavoidable Regularities	31
2.1 Ramsey's theorem	32
2.2 Van der Waerden's theorem	36
2.3 Uniformly recurrent words	41
2.4 Shirshov's theorem	46
2.5 Bounded languages	50
2.6 Power-free words	54
2.7 Bi-ideal sequences	59
2.7.1 Canonical factorizations	61
2.7.2 Bi-ideal sequences and recurrence	66
2.7.3 Some extensions of the Shirshov theorem	72
3. Finiteness Conditions for Semigroups	77
3.1 Preliminaries on semigroups	77
3.2 Finitely generated semigroups	83
3.3 The Burnside problem	88
3.4 Permutation property	90
3.4.1 The weak permutability	96
3.4.2 The ω -permutability	102
3.5 Partial commutations	103
3.6 Chain conditions	105
3.6.1 The J-depth decomposition theorem	109
3.6.2 Minimal conditions on principal right ideals	114
3.6.3 Minimal conditions on principal bi-ideals	116
3.6.4 The McNaughton-Zalcstein and Straubing theorems	123

3.7	Iteration property	127
3.7.1	ω -iteration property	133
3.7.2	Strong periodicity	136
3.8	Permutation and iteration property	137
3.9	Repetitivity	141
3.9.1	Repetitive morphisms and semigroups	141
3.9.2	Strongly repetitive morphisms	142
3.9.3	Uniformly repetitive semigroups	148
4.	Finitely Recognizable Semigroups	153
4.1	The Myhill-Nerode theorem	154
4.2	Finitely recognizable semigroups	158
4.3	The factor semigroup	161
4.4	Rewriting systems	164
4.5	The word problem	166
4.6	On a conjecture of Brzozowski	170
4.6.1	Problems and results	171
4.7	On a conjecture of Brown	175
5.	Regularity Conditions	179
5.1	Uniform conditions	180
5.2	Pumping properties	183
5.3	Permutative property	191
6.	Well Quasi-orders and Regularity	195
6.1	Well quasi-orders	196
6.2	Higman's theorem	199
6.3	The generalized Myhill theorem	203
6.4	Quasi-orders and rewriting systems	206
6.5	A regularity condition for permutable languages	208
6.6	Almost-commutative languages	212
6.7	Copying systems	222
	References	229
	Index	237