Computability & Unsolvability

MARTIN DAVIS

Department of Mathematics and Computer Sciences
Courant Institute of Mathematical Sciences
New York University

DOVER PUBLICATIONS, INC.
NEW YORK

CONTENTS

Preface to the Dover Edition v

Preface to the First Edition vii

Glossary of Special Symbols xiii

Introduction xv

1. Heuristic Remarks on Decision Problems xv
2. Suggestions to the Reader xviii
3. Notational Conventions xix

PART 1
THE GENERAL THEORY OF COMPUTABILITY

Chapter 1. Computable Functions 3

1. Turing Machines 3
2. Computable Functions and Partially Computable Functions . . . 8
3. Some Examples 12
4. Relatively Computable Functions 20

Chapter 2. Operations on Computable Functions 25

1. Preliminary Lemmas 25
2. Composition and Minimalization 36

Chapter 3. Recursive Functions 41

1. Some Classes of Functions 41
2. Finite Sequences of Natural Numbers 43
3. Primitive Recursion 46
4. Primitive Recursive Functions 49
5. Recursive Sets and Predicates 51

Chapter 4. Turing Machines Self-applied 56

1. Arithmetization of the Theory of Turing Machines 56
2. Computability and Recursiveness 62
3. A Universal Turing Machine 64

Chapter 5. Unsolvable Decision Problems 66

1. Semicomputable Predicates 66
2. Decision Problems 69
3. Properties of Semicomputable Predicates 71
4. Recursively Enumerable Sets 73
5. Two Recursively Enumerable Sets 75
6. A Set Which Is Not Recursively Enumerable 77

PART 2
APPLICATIONS OF THE GENERAL THEORY

Chapter 6. Combinatorial Problems 81

1. Combinatorial Systems 81
2. Turing Machines and Semi-Thue Systems 88
3. Thue Systems 94
4. The Word Problem for Semigroups 95
5. Normal Systems and Post Systems 98

Chapter 7. Diophantine Equations 102

1. Hilbert's Tenth Problem 102
2. Arithmetical and Diophantine Predicates 103
3. Arithmetical Representation of Semicomputable Predicates . . . 107

Chapter 8. Mathematical Logic 117

1. Logics 117
2. Incompleteness and Unsolvability Theorems for Logics 120
3. Arithmetical Logics 122
4. First-order Logics 129
5. Partial Propositional Calculi 137

PART 3
FURTHER DEVELOPMENT OF THE GENERAL THEORY

Chapter 9. The Kleene Hierarchy 145

1. The Iteration Theorem 145
2. Some First Applications of the Iteration Theorem 149
3. Predicates, Sets, and Functions 150
4. Strong Reducibility 151
5. Some Classes of Predicates 154
6. A Representation Theorem for P_2^A 156
7. Post's Representation Theorem 159

Chapter 10. Computable Functionals 162

1. Functionals 162
2. Completely Computable Functionals 164
3. Normal Form Theorems 167
4. Partially Computable and Computable Functionals 168
5. Functionals and Relative Recursiveness 169
6. Decision Problems 171
7. The Recursion Theorems 174

Chapter 11. The Classification of Unsolvable Decision Problems . . . 179

1. Reducibility and the Kleene Hierarchy 179
2. Incomparability 180
3. Creative Sets and Simple Sets 183
4. Constructive Ordinals 186
5. Extensions of the Kleene Hierarchy 190

Appendix 1. Some Results from the Elementary Theory of Numbers . . 193

Appendix 2. Hilbert's Tenth Problem Is Unsolvable 199

References 237

Index 243