Numerical Methods, Software, and Analysis

Second Edition

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* Other material skipped in one-semester course

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