

Joram Lindenstrauss Lior Tzafriri

Classical Banach Spaces II

Function Spaces



Springer-Verlag
Berlin Heidelberg New York 1979

Table of Contents

1. Banach Lattices	1
a. Basic Definitions and Results	1
Characterizations of σ -completeness and σ -order continuity. Ideals, bands and band projections. Boolean algebras of projections and cyclic spaces.	
b. Concrete Representation of Banach Lattices	14
Abstract L_p and M spaces. Joint characterizations of $L_p(I)$ and $c_0(\Gamma)$ spaces. The functional representation theorem for order continuous Banach lattices. Köthe function spaces. The Fatou property.	
c. The Structure of Banach Lattices and their Subspaces	31
Property (u). Weak completeness and reflexivity. Existence of unconditional basic sequences.	
d. p -Convexity in Banach Lattices	40
Functional calculus in general Banach lattices. The definition and basic properties of p -convexity and p -concavity. Duality. The p -convexification and p -concavification procedures. Some connections with p -absolutely summing operators. Factorization through L_p spaces.	
e. Uniform Convexity in General Banach Spaces and Related Notions.	59
The definition of the moduli of convexity and smoothness. Duality. Asymptotic behavior of the moduli. The moduli of $L_2(X)$. Convergence of series in uniformly convex and uniformly smooth spaces. The notions of type and cotype. Kahane's theorem. Connections between the moduli and type and cotype.	
f. Uniform Convexity in Banach Lattices and Related Notions	79
Uniformly convex and smooth renormings of a $(p > 1)$ -convex and $(q < \infty)$ -concave Banach lattice. The concepts of upper and lower estimates for disjoint elements. The relations between these notions and those of type, cotype, p -convexity, q -concavity, p -absolutely summing operators, etc. Examples. Two diagrams summarizing the various connections.	
g. The Approximation Property and Banach Lattices.	102
Examples of Banach lattices and of subspaces of l_p , $p \neq 2$, without the B.A.P. The connection between the type and the cotype of a space and the existence of subspaces without the B.A.P. A space different from l_2 all of whose subspaces have the B.A.P.	
2. Rearrangement Invariant Function Spaces	114
a. Basic Definitions, Examples and Results	114
The definition of r, s function spaces on $[0, 1]$ and $[0, \infty)$. Conditional expectations. Interpolation between L_q and L_{∞} .	

b. The Boyd Indices.	129
The definition of Boyd indices. Duality. The Rademacher functions in an r.i. function space on $[0, 1]$. The characterization of Boyd indices in terms of existence of l_p^n 's for all n on disjoint vectors having the same distribution. Operators of weak type (p, q) . Boyd's interpolation theorem.	
c. The Haar and the Trigonometric Systems	150
Basic results on martingales. The unconditionality of the Haar system in $L_p(0, 1)$ spaces ($1 < p < \infty$) and in more general r.i. function spaces on $[0, 1]$. Reproducibility of bases. The boundedness of the Riesz projection and applications.	
d. Some Results on Complemented Subspaces	168
The isomorphism between an r.i. function space X on $[0, 1]$ with non-trivial Boyd indices and the spaces $\mathcal{H}(l_p)$ and $\text{Rad } X$. Complemented subspaces of X with an unconditional basis. Subspaces spanned by a subsequence of the Haar system. R.i. spaces with non-trivial Boyd indices are primary.	
e. Isomorphisms Between r.i. Function Spaces; Uniqueness of the r.i. Structure	181
Isomorphic embeddings. Classification of symmetric basic sequences in r.i. function spaces of type 2. Uniqueness of the r.i. structure for $L_p(0, 1)$ spaces and for $(q < 2)$ -concave r.i. spaces. Other applications.	
f. Applications of the Poisson Process to r.i. Function Spaces	202
The isomorphism between $L_p(0, 1)$ and $L_p(0, \infty) \wedge L_2(0, \infty)$ for $p > 2$. R.i. function spaces in $[0, \infty)$ isomorphic to a given r.i. function space on $[0, 1]$. Isometric embeddings of $L_p(0, 1)$ into $L_q(0, 1)$ for $1 < p < q < 2$ and in other r.i. function spaces. The complementation of the spaces $X_{p,2}$ in $L_p(0, 1)$, $p > 2$ and generalizations.	
g. Interpolation Spaces and their Applications	215
Interpolation pairs. General interpolation spaces and applications to the construction of r.i. function spaces without unique r.i. structure. The Lions-Peetre method of interpolation. Uniform convexity and type in interpolation spaces.	
References	233
Subject Index	239