

SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

OF

SET THEORY

and Related Topics

Second Edition

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SCHAUM'S OUTLINE SERIES

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