

An Introduction to Analysis and Integration Theory

ESTHER R. PHILLIPS

*Professor of Mathematics and Computer Science
Herbert H. Lehman College
of the City University of New York*

With a New Historical Introduction
and Notes by the Author

Dover Publications, Inc.
New York

Contents

PART I The Real Numbers; Metric and Normed Vector Spaces...3

Chapter 0 Preliminaries	3
1. The Algebra of Sets /3	
2. Equivalence Relations and Partial Orderings /7	
3. The Rational Numbers as an Ordered Field /12	
4. Incompleteness of the Rational Numbers /17	
Chapter 1 The Real Number System	25
1. Construction of the Field R /25	
2. Ordering and Completeness of R /28	
3. Theorems About Real Numbers /34	
Chapter 2 Metric and Normed Linear Spaces	49
1. Inequalities /49	
2. Metric and Normed Linear Spaces: Definitions and Examples /54	
3. Topological Properties and Sequences /63	
4. Completion of Metric and Normed Linear Spaces /72	
Chapter 3 Compactness, Continuity, and Connectedness	80
1. Compact Spaces /80	
2. Continuous Functions and Compactness /88	
3. Connected Spaces; The Intermediate Value Theorem /96	
4. The Space $C(X, \mathbb{R})$: Uniform Convergence, Equicontinuity and Arzelà's (Ascoli's) Theorem /104	
Chapter 4 Metric and Normed Linear Spaces: Special Topics... ..	115
1. Uniform Approximation of Functions; The Stone-Weierstrass Theorem /115	
2. Fixed Point Theorems and Contracting Mappings /125	
3. Complete Spaces and the Baire Category Theorem /133	

PART II Introduction...151

Chapter 5 Lebesgue Integrable Functions	155
1. Step Functions and Their Integrals /155	
2. Completion of the Space of Step Functions /158	
3. Null Sets /166	
4. Lebesgue Integrable Functions /172	
Chapter 6 Convergence Theorems and the Riemann Integral ...	181
1. Monotone Sequences of Integrable Functions /182	
2. Fatou's Lemma and the Theorem on Dominated Convergence /189	
3. The Riemann Integral /193	

Chapter 7 Measurable Functions and Measurable Sets	203
1. Measurable Functions /203	
2. Measurable Sets I: Basic Properties /207	
3. Measurable Sets II: More Properties/213	
4. Nonmeasurable Sets and Functions /219	
5. Egoroff's Theorem and Convergence in Measure /222	
Chapter 8 An Alternate Approach to Lebesgue Integration and Measure	230
1. Exterior Measure, Measurable Sets and Functions /230	
2. The Lebesgue Integral (A Second Definition) /235	
An Epilogue to Chapters 5-8 /239	
Chapter 9 Differentiation	241
1. The Derivates of a Function /242	
2. Vitali Coverings /247	
3. Monotone Functions and Functions of Bounded Variation /250	
4. Differentiation and Integration /260	
PART III Banach and Hilbert Spaces...271	
Chapter 10 Banach Spaces, Hilbert Spaces and Fourier Series ..	273
1. L_p Spaces; The Reisz-Fischer Theorem /273	
2. Hilbert Spaces (Geometry) /279	
3. Separable Hilbert Spaces and Orthonormal Sequences /285	
4. Fourier Series /295	
Chapter 11 Linear Functionals	320
1. Bounded Linear Functionals; The Hahn-Banach Theorem /320	
2. Linear Functionals on a Hilbert Space /331	
3. Linear Functionals on L_p Spaces /338	
4. The Stieltjes Integral and Linear Functionals on $C[a, b]$ /346	
PART IV The Daniell Integral and Measure Spaces...361	
Chapter 12 The Daniell Integral and Measure Spaces (I)	363
1. Elementary Functions and Their Integrals /364.	
2. μ -Null Sets /369	
3. Integrable Functions and Convergence Theorems /375	
4. Measurable Functions and Measurable Sets /380	
5. Measure Spaces and the Integral /384	
Chapter 13 Measure Spaces (II): Outer Measures, Signed Measures, Radon-Nikodym Theorem, and Product Measures (Fubini Theorem)	395
1. Outer Measures and the Generation of Measures /396	
2. Lebesgue-Stieltjes Measures and Integrals /402	
3. Signed Measures and the Hahn Decomposition Theorem /412	
4. The Radon-Nikodym Theorem /418	
5. Product Spaces and Fubini's Theorem /431	
Bibliography	441
Symbols	443
Notes to the Dover Edition	447
Index	449