

Reinhard Laubenbacher  
David Pengelley

# Mathematical Expeditions

Chronicles by the Explorers

With 94 Illustrations



Springer Science+Business Media, LLC

Reinhard Laubenbacher  
David Pengelley  
Department of Mathematical Sciences  
New Mexico State University  
Las Cruces, NM 88003-0001  
USA

*Editorial Board*

|                         |                        |                          |
|-------------------------|------------------------|--------------------------|
| S. Axler                | F.W. Gehring           | K.A. Ribet               |
| Mathematics Department  | Mathematics Department | Mathematics Department   |
| San Francisco State     | East Hall              | University of California |
| University              | University of Michigan | at Berkeley              |
| San Francisco, CA 94132 | Ann Arbor, MI 48109    | Berkeley, CA 94720-3840  |
| USA                     | USA                    | USA                      |

FRONT COVER ILLUSTRATIONS: *Background*: Sophie Germain's 1819 letter to Carl F. Gauss about her work on Fermat's Last Theorem. *Foreground* (clockwise from top left): Georg Cantor, Pierre de Fermat, Evariste Galois, Gottfried Leibniz; at center, Nikolai Lobachevsky.

---

Mathematics Subject Classification (1991): 01-01, 04-01, 11-01, 12-01, 26-01, 51-01

---

Library of Congress Cataloging-in-Publication Data

Laubenbacher, Reinhard.

Mathematical expeditions : chronicles by the explorers /

Reinhard Laubenbacher, David Pengelley.

p. cm. — (Undergraduate texts in mathematics. Readings in mathematics)

Includes bibliographical references (p. - ) and index.

ISBN 978-0-387-98433-9 ISBN 978-1-4612-0523-4 (eBook)

DOI 10.1007/978-1-4612-0523-4

I. Mathematics—History—Sources. I. Pengelley, David. II. Title.

III. Series.

QA21.L34 1998

510'.9—dc21

98-3889

Printed on acid-free paper.

© 1999 Springer Science+Business Media New York

Originally published by Springer-Verlag New York in 1999

Softcover reprint of the hardcover 1st edition 1999

All rights reserved. This work may not be translated or copied in whole or in part without the written permission of the publisher (Springer Science+Business Media, LLC), except for brief excerpts in connection with reviews or scholarly analysis. Use in connection with any form of information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed is forbidden. The use of general descriptive names, trade names, trademarks, etc., in this publication, even if the former are not especially identified, is not to be taken as a sign that such names, as understood by the Trade Marks and Merchandise Marks Act, may accordingly be used freely by anyone.

Production managed by Steven Pisano; manufacturing supervised by Thomas M. King.

Typeset from the authors' LaTeX files by The Bartlett Press, Inc., Marietta, GA.

9 8 7 6 5 4 3 2 (Corrected second printing, 2000)

ISBN 978-0-387-98433-9

# Contents

|                                                                     |           |
|---------------------------------------------------------------------|-----------|
| <b>Preface</b>                                                      | <b>v</b>  |
| <b>1 Geometry: The Parallel Postulate</b>                           | <b>1</b>  |
| 1.1 Introduction . . . . .                                          | 1         |
| 1.2 Euclid’s Parallel Postulate . . . . .                           | 18        |
| 1.3 Legendre’s Attempts to Prove the Parallel Postulate . . . . .   | 24        |
| 1.4 Lobachevskian Geometry . . . . .                                | 31        |
| 1.5 Poincaré’s Euclidean Model for Non-Euclidean Geometry . . . . . | 43        |
| <b>2 Set Theory: Taming the Infinite</b>                            | <b>54</b> |
| 2.1 Introduction . . . . .                                          | 54        |
| 2.2 Bolzano’s Paradoxes of the Infinite . . . . .                   | 69        |
| 2.3 Cantor’s Infinite Numbers . . . . .                             | 74        |
| 2.4 Zermelo’s Axiomatization . . . . .                              | 89        |
| <b>3 Analysis: Calculating Areas and Volumes</b>                    | <b>95</b> |
| 3.1 Introduction . . . . .                                          | 95        |
| 3.2 Archimedes’ Quadrature of the Parabola . . . . .                | 108       |
| 3.3 Archimedes’ Method . . . . .                                    | 118       |
| 3.4 Cavalieri Calculates Areas of Higher Parabolas . . . . .        | 123       |
| 3.5 Leibniz’s Fundamental Theorem of Calculus . . . . .             | 129       |
| 3.6 Cauchy’s Rigorization of Calculus . . . . .                     | 138       |
| 3.7 Robinson Resurrects Infinitesimals . . . . .                    | 150       |
| 3.8 Appendix on Infinite Series . . . . .                           | 154       |

|          |                                                                 |            |
|----------|-----------------------------------------------------------------|------------|
| <b>4</b> | <b>Number Theory: Fermat's Last Theorem</b>                     | <b>156</b> |
| 4.1      | Introduction . . . . .                                          | 156        |
| 4.2      | Euclid's Classification of Pythagorean Triples . . . . .        | 172        |
| 4.3      | Euler's Solution for Exponent Four . . . . .                    | 179        |
| 4.4      | Germain's General Approach . . . . .                            | 185        |
| 4.5      | Kummer and the Dawn of Algebraic Number Theory . . . . .        | 193        |
| 4.6      | Appendix on Congruences . . . . .                               | 199        |
| <b>5</b> | <b>Algebra: The Search for an Elusive Formula</b>               | <b>204</b> |
| 5.1      | Introduction . . . . .                                          | 204        |
| 5.2      | Euclid's Application of Areas and Quadratic Equations . . . . . | 219        |
| 5.3      | Cardano's Solution of the Cubic . . . . .                       | 224        |
| 5.4      | Lagrange's Theory of Equations . . . . .                        | 233        |
| 5.5      | Galois Ends the Story . . . . .                                 | 247        |
|          | <b>References</b>                                               | <b>259</b> |
|          | <b>Credits</b>                                                  | <b>269</b> |
|          | <b>Index</b>                                                    | <b>271</b> |