

---

# **COMPLEX VARIABLES AND APPLICATIONS**

---

**Sixth Edition**

**James Ward Brown**

*Professor of Mathematics  
The University of Michigan—Dearborn*

**Ruel V. Churchill**

*Late Professor of Mathematics  
The University of Michigan*

**McGraw-Hill, Inc.**

New York St. Louis San Francisco Auckland Bogotá  
Caracas Lisbon London Madrid Mexico City Milan  
Montreal New Delhi San Juan Singapore  
Sydney Tokyo Toronto

---

# CONTENTS

---

Preface	xv
<b>1 Complex Numbers</b>	1
1. Sums and Products	1
2. Algebraic Properties	2
3. Moduli and Conjugates	6
4. Triangle Inequality	9
5. Polar Coordinates and Euler's Formula	12
6. Products and Quotients in Exponential Form	14
7. Roots of Complex Numbers	19
8. Regions in the Complex Plane	23
<b>2 Analytic Functions</b>	26
9. Functions of a Complex Variable	26
10. Mappings	28
11. Limits	33
12. Theorems on Limits	35
13. Limits Involving the Point at Infinity	38
14. Continuity	40
15. Derivatives	43
16. Differentiation Formulas	45
17. Cauchy-Riemann Equations	48
18. Sufficient Conditions for Differentiability	50
19. Polar Coordinates	52
20. Analytic Functions	55
21. Reflection Principle	57
22. Harmonic Functions	59
<b>3 Elementary Functions</b>	65
23. The Exponential Function	65
24. Trigonometric Functions	69
25. Hyperbolic Functions	72
26. The Logarithmic Function and Its Branches	75
27. Some Identities Involving Logarithms	78
28. Complex Exponents	81
29. Inverse Trigonometric and Hyperbolic Functions	83

<b>4 Integrals</b>	86
30. Complex-Valued Functions $w(t)$	86
31. Contours	89
32. Contour Integrals	95
33. Examples	97
34. Antiderivatives	104
35. Examples	107
36. Cauchy-Goursat Theorem	110
37. Proof of the Theorem	112
38. Simply and Multiply Connected Domains	116
39. Cauchy Integral Formula	123
40. Derivatives of Analytic Functions	125
41. Liouville's Theorem and the Fundamental Theorem of Algebra	130
42. Maximum Moduli of Functions	132
<b>5 Series</b>	138
43. Convergence of Sequences and Series	138
44. Taylor Series	143
45. Examples	146
46. Laurent Series	150
47. Examples	154
48. Absolute and Uniform Convergence of Power Series	159
49. Integration and Differentiation of Power Series	164
50. Uniqueness of Series Representations	167
51. Multiplication and Division of Power Series	169
52. Analytic Continuation	176
<b>6 Residues and Poles</b>	180
53. Residues	180
54. Residue Theorems	183
55. The Three Types of Isolated Singular Points	186
56. Residues at Poles	190
57. Zeros and Poles of Order $m$	193
58. Conditions under Which $f(z) = 0$	199
59. Behavior of $f$ Near Removable and Essential Singular Points	201
<b>7 Applications of Residues</b>	204
60. Evaluation of Improper Integrals	204
61. Improper Integrals Involving Sines and Cosines	210
62. Definite Integrals Involving Sines and Cosines	217
63. Indented Paths	219
64. Integration along a Branch Cut	224
65. Argument Principle and Rouché's Theorem	228
66. Inverse Laplace Transforms	235
67. Examples	238
<b>8 Mapping by Elementary Functions</b>	245
68. Linear Transformations	245
69. The Transformation $w = 1/z$	247
70. Linear Fractional Transformations	251
71. An Implicit Form	254

72.	Mappings of the Upper Half Plane	256
73.	Exponential and Logarithmic Transformations	260
74.	The Transformation $w = \sin z$	263
75.	Mappings by Branches of $z^{1/2}$	268
76.	Square Roots of Polynomials	271
77.	Riemann Surfaces	276
78.	Surfaces for Related Functions	279
<b>9</b>	<b>Conformal Mapping</b>	283
79.	Preservation of Angles	283
80.	Further Properties	285
81.	Harmonic Conjugates	290
82.	Transformations of Harmonic Functions	292
83.	Transformations of Boundary Conditions	294
<b>10</b>	<b>Applications of Conformal Mapping</b>	299
84.	Steady Temperatures	299
85.	Steady Temperatures in a Half Plane	301
86.	A Related Problem	303
87.	Temperatures in a Quadrant	305
88.	Electrostatic Potential	310
89.	Potential in a Cylindrical Space	311
90.	Two-Dimensional Fluid Flow	315
91.	The Stream Function	317
92.	Flows around a Corner and around a Cylinder	319
<b>11</b>	<b>The Schwarz-Christoffel Transformation</b>	326
93.	Mapping the Real Axis onto a Polygon	326
94.	Schwarz-Christoffel Transformation	328
95.	Triangles and Rectangles	330
96.	Degenerate Polygons	334
97.	Fluid Flow in a Channel through a Slit	339
98.	Flow in a Channel with an Offset	341
99.	Electrostatic Potential about an Edge of a Conducting Plate	344
<b>12</b>	<b>Integral Formulas of the Poisson Type</b>	349
100.	Poisson Integral Formula	349
101.	Dirichlet Problem for a Disk	351
102.	Related Boundary Value Problems	354
103.	Schwarz Integral Formula	358
104.	Dirichlet Problem for a Half Plane	360
105.	Neumann Problem for a Disk	362
106.	Neumann Problem for a Half Plane	363
<b>Appendixes</b>		367
1.	Bibliography	367
2.	Table of Transformations of Regions	370
<b>Index</b>		379