

Wavelets

Algorithms & Applications

Yves Meyer
CEREMADE
and

Institut Universitaire de France

Translated and Revised by

Robert D. Ryan

Office of Naval Research
European Office

Society for Industrial and Applied Mathematics
Philadelphia 1993

siam

Contents

Translator's Foreword	ix
-----------------------	----

Preface	xi
---------	----

CHAPTER 1. Signals and Wavelets	
1.1 What Is a Signal?	1
1.2 The Goals of Signal and Image Processing	2
1.3 Stationary Signals, Transient Signals, and Adaptive Coding Algorithms	4
1.4 Grossmann-Morlet Time-Scale Wavelets	5
1.5 Time-Frequency Wavelets from Gabor to Malvar	6
1.6 Optimal Algorithms in Signal Processing	7
1.7 Optimal Representation, according to Marr	9
1.8 Terminology	10
1.9 Reader's Guide	11
CHAPTER 2. Wavelets From a Historical Perspective	
2.1 Introduction	13
2.2 From Fourier (1807) to Hoor (1909), Frequency Analysis Becomes Scale Analysis	14
2.3 New Directions of the 1930s: Lévy and Brownian Motion	18
2.4 New Directions of the 1930s: Littlewood and Paley	19
2.5 New Directions of the 1930s: The Franklin System	21
2.6 New Directions of the 1930s: The Wavelets of Lusin	22
2.7 Atomic Decompositions, from 1960 to 1980	24
2.8 Strömberg's Wavelets	26
2.9 A First Synthesis: Wavelet Analysis	27
2.10 The Advent of Signal Processing	30
2.11 Conclusions	31
CHAPTER 3. Quadrature Mirror Filters	
3.1 Introduction	33
3.2 Subband Coding: The Case of Ideal Filters	34
3.3 Quadrature Mirror Filters	36
3.4 The Trend and Fluctuation	38
3.5 The Time-Scale Algorithm of Mallat and the Time-Frequency Algorithm of Galand	39
3.6 Trends and Fluctuations with Orthonormal Wavelet Bases	40
3.7 Convergence to Wavelets	42
3.8 The Wavelets of Daubechies	43
3.9 Conclusions	44

CHAPTER 4.	Pyramid Algorithms for Numerical Image Processing	
4.1	Introduction	45
4.2	The Pyramid Algorithms of Burt and Adelson	46
4.3	Examples of Pyramid Algorithms	50
4.4	Pyramid Algorithms and Image Compression	51
4.5	Pyramid Algorithms and Multiresolution Analysis	53
4.6	The Orthogonal Pyramids and Wavelets	55
4.7	Bi-orthogonal Wavelets	59
CHAPTER 5.	Time-Frequency Analysis for Signal Processing	
5.1	Introduction	63
5.2	The Time-Frequency Plane	66
5.3	The Wigner-Ville Transform	66
5.4	The Computation of Certain Wigner-Ville Transforms	67
5.5	The Wigner-Ville Transform and Pseudodifferential Calculus	69
5.6	The Wigner-Ville Transform and Instantaneous Frequency	70
5.7	The Wigner-Ville Transform of Asymptotic Signals	72
5.8	Returns to the Problem of Optimal Decomposition in Time-Frequency Atoms	73
CHAPTER 6.	Time-Frequency Algorithms Using Malvar's Wavelets	
6.1	Introduction	75
6.2	Malvar Wavelets: A Historical Perspective	76
6.3	Windows with Variable Lengths	77
6.4	Malvar Wavelets and Time-Scale Wavelets	80
6.5	Adaptive Segmentation and the Split-and-Merge Algorithm	81
6.6	The Entropy of a Vector with Respect to an Orthonormal Basis	83
6.7	The Algorithm for Finding the Optimal Malvar Basis	83
6.8	An Example Where This Algorithm Works	85
6.9	The Discrete Case	86
CHAPTER 7.	Time-Frequency Analysis and Wavelet Packets	
7.1	Heuristic Considerations	89
7.2	The Definition of Basic Wavelet Packets	92
7.3	General Wavelet Packets	95
7.4	Splitting Algorithms	97
7.5	Conclusions	98
CHAPTER 8.	Computer Vision and Human Vision	
8.1	Marr's Program	101
8.2	The Theory of Zero-Crossings	104
8.3	A Counterexample to Marr's Conjecture	105
8.4	Mallat's Conjecture	106
8.5	The Two-Dimensional Version of Mallat's Algorithm	109
8.6	Conclusions	110
CHAPTER 9.	Wavelets and Fractals	
9.1	Introduction	111
9.2	The Weierstrass Function	111
9.3	The Determination of Regular Points in a Fractal Background	113
9.4	Study of the Riemann Function	115
9.5	Conclusions	117

CHAPTER 10.	Wavelets and Turbulence	
10.1	Introduction	119
10.2	The Statistical Theory of Turbulence and Fourier Analysis	119
10.3	Verification of the Hypothesis of Pinski and Frisch	120
10.4	Farge's Experiments	121
10.5	Numerical Approaches to Turbulence	122
CHAPTER 11.	Wavelets and the Study of Distant Galaxies	
11.1	Introduction	125
11.2	The New Telescopes	125
11.3	The Hierarchical Organization of the Galaxies and the Creation of the Universe	126
11.4	The Multifractal Approach to the Universe	126
11.5	The Advent of Wavelets	126
	Index	129