

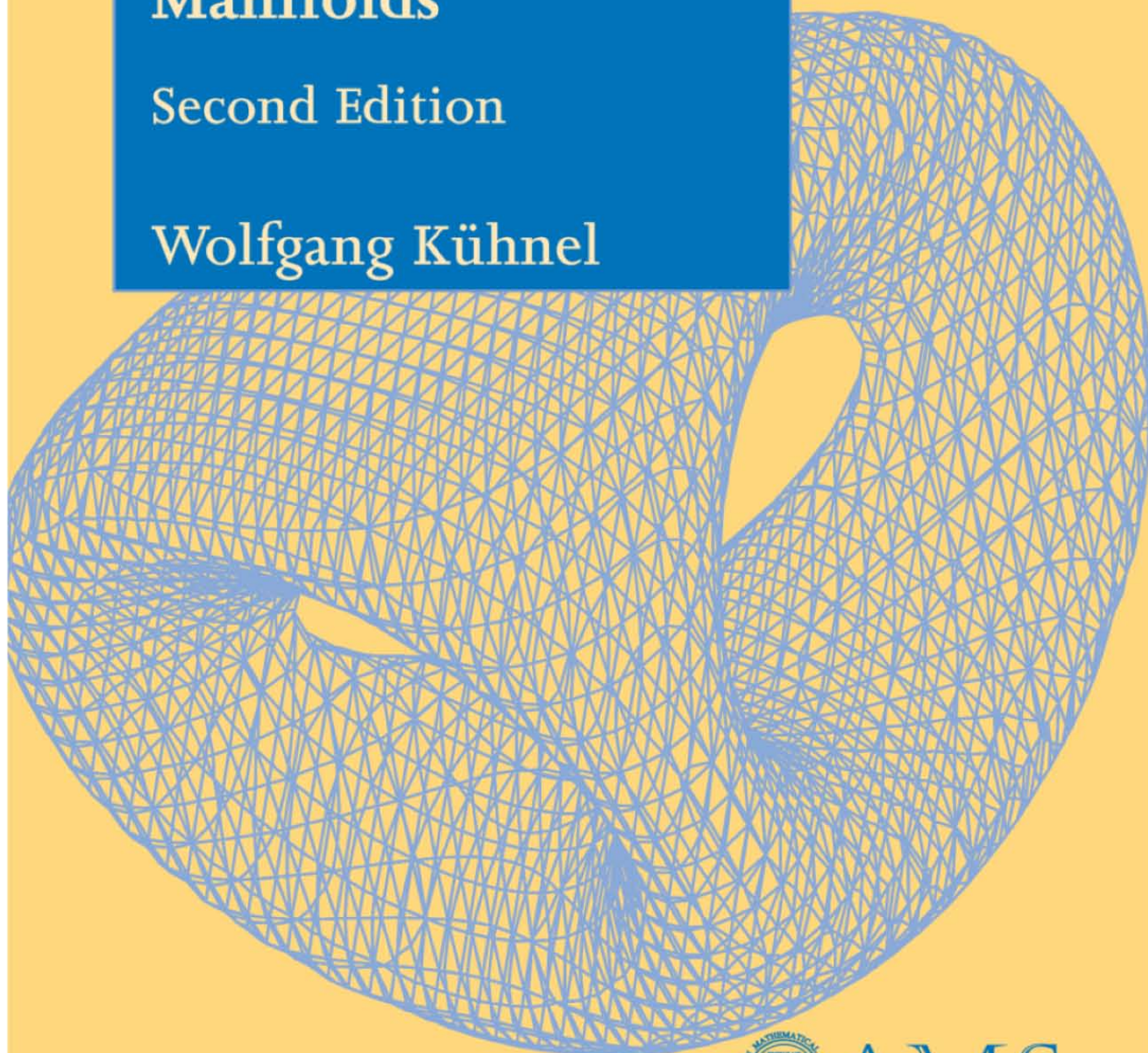
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Volume 16

Differential Geometry

Curves – Surfaces –
Manifolds

Second Edition

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AMS

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Contents

Preface to the English Edition	ix
Preface to the German Edition	xi
Chapter 1. Notations and Prerequisites from Analysis	1
Chapter 2. Curves in $\mathbb{R}^n$	7
2A Frenet curves in $\mathbb{R}^n$	7
2B Plane curves and space curves	14
2C Relations between the curvature and the torsion	20
2D The Frenet equations and the fundamental theorem of the local theory of curves	27
2E Curves in Minkowski space $\mathbb{R}_1^3$	33
2F The global theory of curves	37
Exercises	49
Chapter 3. The Local Theory of Surfaces	55
3A Surface elements and the first fundamental form	56
3B The Gauss map and the curvature of surfaces	66
3C Surfaces of rotation and ruled surfaces	78
3D Minimal surfaces	97
3E Surfaces in Minkowski space $\mathbb{R}_1^3$	114

<u>3F Hypersurfaces in $\mathbb{R}^{n+1}$</u>	124
Exercises	127
<u>Chapter 4. The Intrinsic Geometry of Surfaces</u>	133
<u>4A The covariant derivative</u>	134
<u>4B Parallel displacement and geodesics</u>	140
<u>4C The Gaussian equation and the Theorema Egregium</u>	146
<u>4D The fundamental theorem of the local theory of surfaces</u>	152
<u>4E The Gaussian curvature in special parameters</u>	158
<u>4F The Gauss-Bonnet Theorem</u>	166
<u>4G Selected topics in the global theory of surfaces</u>	181
Exercises	194
<u>Chapter 5. Riemannian Manifolds</u>	201
<u>5A The notion of a manifold</u>	202
5B The tangent space	209
5C Riemannian metrics	217
<u>5D The Riemannian connection</u>	223
Exercises	233
<u>Chapter 6. The Curvature Tensor</u>	239
6A Tensors	239
6B The sectional curvature	248
6C The Ricci tensor and the Einstein tensor	256
Exercises	266
<u>Chapter 7. Spaces of Constant Curvature</u>	269
<u>7A Hyperbolic space</u>	270
7B Geodesics and Jacobi fields	280
7C The space form problem	296
<u>7D Three-dimensional Euclidean and spherical space forms</u>	302
Exercises	311

Chapter 8. Einstein Spaces	315
8A The variation of the Hilbert-Einstein functional	318
8B The Einstein field equations	326
8C Homogeneous Einstein spaces	331
8D The decomposition of the curvature tensor	336
8E The Weyl tensor	346
8F Duality for four-manifolds and Petrov types	355
Exercises	364
Bibliography	367
List of notation	371
Index	375